Grade 5 solutions

1. **Answer:** 15. We must divide 30 by 2 to obtain the answer.
2. **Answer:** 2. We can only change the order of AAS, so the different orderings are SAA and ASA.
3. **Answer:** 24. 4 more people went off the bus than came onto the bus. Therefore, the total number of people decreased by 4 from 28 to 24.
4. **Answer:** 1. I have four-fifth left of my money. Multiply $\frac{4}{5}$ by 1.25 to obtain 1.
5. **Answer:** 15. To find my age, plug in 13 for Ben, which then leads to 18 for Jae and then 15 for me.
6. **Answer:** 96. First the stock rises to $120 by increasing $20. To find the new price, multiply 120 by 0.8 to obtain the answer.
7. **Answer:** 25. A simple case of adding and subtracting the correct numbers. Add 35, subtract 15 and add 5 to arrive at the final answer
8. **Answer:** 16. Mr. Holbrook buys 80 dozen donuts, costing $800. Divide that by $50 to obtain the number of students that need to attend
9. **Answer**: 9. There are ten integers that satisfy these conditions between 1 and 100 inclusive, so excluding one, there are nine integers.
10. **Answer:** 1700. Add the fractions of China and India to obtain $\frac{17}{24}$, which then is multiplied with 2400 to obtain the answer.
11. **Answer:** 8.709. Careful to line up the decimal points.
12. **Answer:** 175. Summing from 2 to 8, we may use one of two formulae: $S_n = \frac{a_1 + a_n}{2} \times n$, in which n represents the number of terms in the arithmetic sequence, a represents the terms, and S represents the sums. The other approach is to find the sum of the first 8 integers and subtract by one. The formula for summing the first n integers is $\frac{n(n+1)}{2}$. Try to derive the formula as an exercise. Do not forget to multiply by 5 to get the final result.
13. **Answer:** $\frac{1}{2}$. There are ten letters and 5 consonants.
14. **Answer:** 2. Mouse A will each one-sixth of the bag in one hour and mouse B will eat one-third of the bag in one hour. Together, they will eat half a bag in an hour. Therefore, they will finish the bag in two hours
15. **Answer:** $\frac{65}{72}$. Add the fractions to get the answer.
16. **Answer:** 172800. Take the 1 day and convert them into seconds by multiplying by 24, 60, and 60. Then, multiply by two to obtain the number of blips.
17. **Answer:** 435. Add 250 to 60 and split the registration fee in four and add that to 310. Another approach is to calculate the total costs, by multiplying the ticket and hotel costs, and divide by 4.
18. **Answer:** 23. A problem here involving PEMDAS. Make sure you are doing things in the correct order.
19. **Answer:** 93. Let the number of rabbits be a and the number of chicken be b. Then set up the following system of equations $\begin{matrix} 4a + 2b = 1422 \\ a + b = 402 \end{matrix}$. Multiply the second equation by four and subtract the first equation from it. Divide by 2 to obtain the answer.
20. **Answer:** 30. Add 7 inches to Yao's height and we find Kevin wants to be eight foot tall. To reach that height, he needs another 2 feet 6 inches. Convert the feet to inches to arrive at the final answer.

21. **Answer:** 40. Let t be the time that has passed. The temperature of the room is 50+0.5t while the pie is 300-4t. Set two times the room temperature equal to the pie temperature, and solve for t.
22. **Answer:** 49. Substitute 7 Fan for Tom in the equation "14 Tom is equal to 1 Husk." Then, substitute the 2 Harry for 1 Husk in the same equation. Therefore, 98 Fan equal 2 Harry, and 49 Fan is equal to one Harry.
23. **Answer:** 21. The number of questions in the database is 7 times 24. Now Dick has decided to do 2 questions, the group uses up 8 questions per day. Dividing 168 by 8, we obtain the answer.
24. **Answer:** $\frac{11}{7}$. Adding the total bases in the game with the 10 bases before, Jon Chu has 22 total bases in 14 at-bats. Divide through to find the answer.
25. **Answer:** 25. Solver beware. It may seem that Gary goes 2 centimeters everyday, and that he reaches 52 centimeters after 26 days. However, Gary only slips at night if he is still in the hole. During the 25^{th} day, Gary reaches 51 centimeters from the 48 centimeter height he began the day on.
26. **Answer:** 50. A simple case of Principle of Inclusion-Exclusion. We need to find how many we over-counted. We know that a total of 550 people attended the two events, but only 500 people attended at least one event. That means that 550-500=50 people were counted twice, because they went on both trips.
27. **Answer:** 79. Arrange the given conditions into three equations and add them. This will result in two bags of each costing 158 cents. Divide by two to find the answer.
28. **Answer:** 156. Square 1.6 and multiply the answer by 100 and then subtract 100 to obtain the answer
29. **Answer:** 16π . $24 = 6s^2$. So, s=2.
30. **Answer:** 3. Since a - b is not equal to 0, we multiply both sides by a - b. Then we distribute and collect like terms, with a's on one side and b's on the other. Then divide both sides by b (b is clearly not 0 from the original equation) to obtain a value for $\frac{a}{b}$
31. **Answer:** $\frac{1}{6}$. Without loss of generality, let Tom line up his horses in a line in the order A, C, and E. Chris then must randomly pick an order for his horses, which totals 6 possibilities. Chris only wins in one of these cases, when F faces A, B faces C, and D faces E. This problem was inspired by an old Chinese story, where a person with weaker horses still managed to win two out of three races.
32. **Answer:** 12. $72 = 2^3 \times 3^2$. Adding one to every exponent and multiplying them yields the correct answer. To derive the formula, write out the powers of 2 up to the power in the expression in one set and the powers of 3 in another (include 1 for both sets). Draw lines from every power of 2 to every power of three. Multiply the numbers connected by each line, and they should represent all the positive factors of 72. Notice that nothing repeats.
33. **Answer:** 7. Let's look at the possible number of packs of five we can have. We can have 12 packs with no packs of two, 10 packs with 5 packs of two and so on until we have 0 packs of five with 30 packs of two. We cannot have an odd number of packs of five, because we can no longer reach exactly sixty. Therefore, there are 7 possibilities.
34. **Answer:** 210. The smallest three funny numbers are made up of the primes 2, 3, 5, 7. The product of the four numbers is 210.
35. **Answer:** $\frac{3}{8}$. $\frac{\sqrt[4]{3}}{\sqrt[8]{3}} = \frac{3^y}{\sqrt[4]{3}}$. Dividing powers with the same base, we must subtract the exponents. We now set $\frac{1}{4} - \frac{1}{8} = y - \frac{1}{4}$ and solve for y.

36. **Answer:** 14. First find the total distance of the round trip and the total time of the trip. Then just divide the distance by the time. Note that you cannot average the speeds of the two parts. The answer to this question is a *weighted* average of these two speeds.
37. **Answer:** $\frac{2}{3}$. A hidden geometric series problem. The duel has (1-$\frac{1}{3}$)(1- $\frac{1}{4}$)= $\frac{1}{2}$ chance of going to an extra round. Burr has a $\frac{1}{3}$ chance of winning the duel on his first shot. If both men miss, he now has a $\frac{1}{2}\times\frac{1}{3}$ chance of winning. If both miss again, Burr now has $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{3}$ of winning, and so on and so forth. The geometric series $1+\frac{1}{2}+\frac{1}{4}+\ldots$ equals 2. Multiplying by $\frac{1}{3}$, we obtain that Burr has a two-thirds chance of winning the duel.
38. **Answer:** 10. Add 1 to both sides and factor the left side into (x+1)(y+1)=35. The only values of x and y that work are 4 and 6.
39. **Answer:** 19/12. For the angle CAB to be the largest angle, side BC, (x+7), must be the longest side. However, for the triangle to be non-degenerate, side BC cannot be longer than the sum of the other two sides. So the two inequalities that must be satisfied are $\begin{matrix} x+7>4x \\ 4x+x+4>x+7 \end{matrix}$ (note that x+7 is always greater than x+4). Therefore the two bounds are $\frac{7}{3}$ and $\frac{3}{4}$.
40. **Answer:** 99. We want to find all 2 and 3-digit palindromes. 2-digit palindromes are determined by their first digit, and 3-digit palindromes are determined by their first 2 digits. However, note the fact that zero cannot be the lead digit in either case. Therefore, we can have $9+9\times10$ =99 palindromes.
41. **Answer:** 31. The least common multiple of 60 and 62 is 1860 seconds. Dividing by 60, we obtain the answer.
42. **Answer:** 2005. In every game, exactly one participant is eliminated. Therefore, we need 2005 games to eliminate all but the champion.
43. **Answer:** 34. The amount of incorrectly graded questions, on average, is $40\times\frac{1}{20}\times\frac{1}{2}=1.$ Therefore they expect to grade 39 questions correctly and 1 incorrectly.
44. **Answer:** $\frac{2}{3}$. The height of the container is 6r and the radius is r. The ratio we are looking for is $\frac{3\times\frac{4}{3}\pi r^3}{\pi r^2\times 6r}$ which cancels out to the final answer.
45. **Answer:** 16. Let v be the velocity of Josh walking. From the given information, the total distance is $\frac{40}{3}\times v+\frac{20}{3}\times 2v=\frac{80}{3}v$. Let t be the time Josh takes to reach school on the second day. $\frac{2}{3}\times 2v\times t+v\times\frac{t}{3}=\frac{80}{3}v$. Solve for t to obtain the answer.
46. **Answer:** 2. Draw a line parallel to AC through K. Let the new point be D. Triangles KBD and ABC are similar, so KB=KD. LC is the midline of triangle KMD. Therefore LC is one half of both KB and KD.

47. **Answer:** 2. Let a be the number of 10 pence coins, b be the number of 20 pence coins and c be the number of 50 pence coins. Take the system of equations: $10a + 20b + 50c = 500$ $a + b + c = 20$ and multiply the second by 20. Then subtract the second from the first. This will yield the equation 3c-a=10. The only pair of integers that satisfies the given conditions are a=2 and c=4.

48. **Answer:** $\sqrt{26}$. Construct the lines from T and S that are perpendicular from the legs. Let point E be the point on Leg RQ such that it is on the end of the line extending from S. Let EQ and ES have lengths x cm and y cm, respectively. Let Point A be the other end of the line extending from T to leg PR. Triangles PAT, TCS, and SEQ are congruent (ASA). Let C be the intersection of the line from S to leg PR and line from T to leg RQ.

$\therefore$ AT = CS = EQ = x

$\therefore$ D and E trisect RQ

Similarly, A and B trisect PR. In triangle SRE, $RE^2 + ES^2 = 7^2$

$\therefore$ $4x^2 + y^2 = 49$

Similarly, from triangle ATR,

$x^2 + 4y^2 = 81$

Adding the two equations,

$5x^2 + 5y^2 = 130$

$x^2 + y^2 = 26$

But $ST^2 = SQ^2 = x^2 + y^2 = 26$

$\therefore$ ST = $\sqrt{26}$

49. **Answer:** 9. Set up an equation between the first and second child:

$1000 + \frac{(x-1000)}{10} = 1000 + \frac{1}{10}(x - 1000 - \frac{1}{10}(x-1000) - 2000)$ and solve for x, which is the amount of money Bernoulli has. Then, find the amount each child receives, namely 9000, and divide x by 9000.

50. **Answer:** 7. The tortoise can answer 0 out of 1 the first day and $\frac{7}{11}$ the second day. This question once again involves the concept of weighted averages. This question demonstrates Simpson's Paradox, which states that even though a person may have a lower percentage of correct answers on both days, he can still have more correct answers.