8th Grade Competition

Bergen County Academies Math Competition
21 October 2007

1. A student is compiling 250 questions for a math competition. She asked each student to write at least 5 questions with solutions. She accepted a question if and only if it came with a solution and was unique. Out of 125 math team members, only 15 wrote questions. One student wrote 42 questions with solutions. One student wrote 12 questions with solutions. Seven students gave 5 questions each without solutions. Six students gave 2 questions each with solutions. However, she received 3 repeats. How many questions did she have to produce herself?

2. $-51 - (-52) = ?$

3. Joyce is making a classroom model of the solar system. Mercury, the smallest planet, has a diameter of $3,000$ miles. Jupiter, the largest planet, has a diameter of $88,700$ miles. She decided to use a pea measuring 0.5 cm in diameter as the Mercury model. Using the same scale, what size, to the nearest whole centimeter, should the diameter of the Jupiter model be?

4. $\frac{2}{5} = \frac{8}{?}$

5. Pavel wants to buy a CD player that costs $56 including tax. He gets $10 a week for his allowance. He spends $3.50 a week and saves the rest. How many weeks will it take him to save enough money to buy the CD player?

6. If $v + w = w$, what is the value of v?

7. Vincent runs at a constant rate of 7 meters per second. At the beginning of a race, he is 350 meters from the finish line. How many seconds will it take Vincent to reach the finish line?

8. A telephone call to India costs $0.29 per minute. How long was a telephone call that cost $1.45?

9. In how many ways can 4 socks be chosen from a drawer containing 9 socks of different colors?

10. What is the number halfway between $\frac{1}{11}$ and $\frac{1}{7}$?

11. A certain book has 500 pages numbered $1, 2, 3$, and so on. How many times does the digit 1 appear in the page numbers?

12. Marina spent two thirds of her money. Then she lost two thirds of the money that was left. Four dollars remained. How much money did Marina have in the beginning?

13. What is the least common multiple of the numbers $6, 9$, and 25?

14. When Ben has a "sharpie battle," he has a $1/3$ chance of poking his opponent's arm and a $1/5$ chance of poking their neck (neither affects the other). When he faces Yoonjoo, what is the probability that, per shot, he pokes her arm, her neck, and her arm again in that order?

15. Robert has two watches, one which loses 6 seconds every 24 hours and one which gains 1 second per hour. He sets both of them to the correct time at $6:00$ p.m. How many hours will pass before the positive difference between the time shown on both watches is 4 hours?

16. Ashley is 18 years old and Scott is twice her age. If Ashley's mom is 10 years older than Scott, how old is Ashley's mom?

17. $36^{1/2} = ?$

18. Matt has a magic number basket. The only numbers that can be placed in the basket are numbers that have two, three, or four digits, all the digits must be odd, and the digits must increase from left to right. How many numbers can Matt place in the basket?

19. The arithmetic mean of 10 numbers is what percent of the sum of the same 10 numbers?

20. Twenty chess players hold a tournament in which each player plays one game with each of the other players. How many games are played altogether?

21. In an isosceles triangle, the perpendicular bisector of one leg passes through the midpoint of the base. If the length of this leg is 10, how long is the base in simplest terms?

22. Find the value of n if: $\log_2 \log_3 \log_4 2^n = 2$

23. In May, the price of a pair of jeans was 200% of its wholesale cost. In June, the price was reduced by 15%. After another 20% discount in July, the jeans cost \$40.80 . What was the wholesale cost of the jeans?

24. Find all ordered pairs of positive integers (a, b) such that $a < b$ and $\sqrt{10 + \sqrt{84}} = \sqrt{a} + \sqrt{b}$

25. Find the sum of the first 50 terms in the arithmetic sequence $2, 6, 10, 14, \ldots$

26. Trapezoid $ABCD$ is such that $AB = 43$ cm is parallel to $CD = 207$ cm. $m\angle ADC = 45$ degrees. If the area of the trapezoid is 10500 cm^2, how long is line segment BC, in cm?

27. The Bergen County Academies is hosting a school-wide Super Smash Brothers tournament. Twenty-four students signed up for the tournament. Each player plays one other player in each round, and the winner advances to the next round. Elimination games continue until there is one winner. How many games must be played to find the winner?

28. What is the smallest value of k for which $4x^2 + kx - 14x + 25$ is a perfect square for all integer x?

29. At the science fair, Veena presents her project every 15 minutes. She gets 1 visitor at $1:00$. This person is so enthralled, he tells 2 different people to see it at her next presentation. Each of these people has the same reaction, and each tell 2 different people to see it at the next presentation. If this pattern continues, with every person who sees the project making 2 other people see it the next time it is presented, and no one else discovers it, how many people will be present at her last presentation, at $2:30$?

30. If two distinct numbers are selected at random from the first 13 prime numbers, what is the probability that their sum is even?

31. Two people stand back to back next to the rails in a small railway station. As the head of the express train that passes the station reaches them, they start to walk away from each other parallel to the rails. As the tail of the train reaches each of them, they stop, having walked 30m and 40m respectively. If they both walked at the same constant speed and the train moved at a constant speed, how long was the train?.

32. Lucy arranges 45 chords of a given circle so as to yield the maximum number of points of intersection of two chords inside the circle. What is the maximum number?

33. An absentminded bank teller switches the dollars and cents when he cashed a check for Hernando, giving him dollars instead of cents, and cents instead of dollars. After buying a five cent newspaper, Hernando discovered he had left exactly twice as much as his original check. What was the amount of the check?

34. If the integer $(5^3) * (3^b)$ is divisble by only 20 positive integers, then b is equal to:

35. Let a, b and c represent the lengths of the sides of a right triangle with $b < c < a$. If $L = 8a^2 + 20b^2 + 20c^2$, find the value of $L/(b^2 + c^2)$.

36. A convex polygon has n sides and $15n$ diagonals. Find the value of n.

37. In the geometric sequence $1, 2, 4, 8, ...$, the nth term is 1024. Find n.

38. What is the surface area of a cube with side length 1.5?

39. The measure of the supplement of an angle is 4 times the measure of its complement. Find the sum, in degrees, of the measure of the angle.

40. How many terminating zeroes does the number 125! have?

41. Solve for x : $\frac{1}{10^1} + \frac{1}{10^2} + \frac{1}{10^3} + \frac{1}{10^4} + \frac{1}{10^5} + \frac{1}{10^6} + \frac{1}{10^7} + \frac{1}{10^8} = \frac{x}{10^8}$.

42. Consider a rectangle $ABCD$ where $AB = 6cm$, and $BC = 9$ cm. Let M be a point on the segment AB. In cm^2, what is the area of $\triangle MCD$?

43. What is the greatest number of right triangular sections, each with base $= 3$ inches and height $= 5$ inches, that can be cut from a rectangular piece of paper measuring 55 inches by 21 inches?

44. When each side of a square is increased by 2 feet, the area is increased by 24 square feet. By how many feet does each side of the original square have to be decreased in order to decrease the area of the original square by 24 square feet?

45. How many points, no three of which are collinear, determine 91 lines?

46. $\frac{\sin 10 \cos 10 \tan 10 \cot 10 \sec 10 \csc 10}{\sin 20 \cos 20 \tan 20 \cot 20 \sec 20 \csc 20} = ?$

47. In the coordinate plane, the graph of the function f is the point $(3, -4)$. What is the distance between the graph of f and the graph of its inverse, $f^{-}1$?

48. A sequence of number $a_1, a_2, a_3, a_4, \ldots$ is defined by $a_1 = 7, a_2 = -6$ and $a_n = a_{n-1} - a_{n-2}$. What is the sum of the first 1587 terms of the sequence?

49. $f(x) = x^2 + x + 11$. x is chosen from $\{1, 2, \ldots, 9\}$. What is the probability that $f(x)$ is prime?

50. $1/2 + 1/6 + 1/12 + 1/20 + \ldots + 1/4830 = ?$