4th Grade Competition

Bergen County Academies Math Competition
21 October 2007

1. A student has to compile 250 questions for a math competition. She asked each student on the math team to write 1 question. If there are 125 students on the team, not including herself, how many extra questions does she have to produce?
   Answer: 125
   If each student was supposed to write 1 question and there are 125 students, then she should have received 125 questions total. However, since she needs 250 questions, she needs to write $250 - 125 = 125$ extra questions.

2. $2 + 0 - 0 + 7 =?$
   Answer: 9
   $2 + 0 = 2; 2 - 0 = 2; 2 + 7 = 9$

3. How many \$0.30 packs of gum can I purchase with \$3.00?
   Answer: 10
   I can purchase $\frac{\$3.00}{\$0.30} = 10$ packs of gum.

4. Tom, Dick, and Harry have 100 marbles. Tom has as many marbles as Dick. Harry has 40 marbles. How many marbles does Dick have?
   Answer: 30
   Tom and Dick have $100 - 40 = 60$ marbles, so Dick has $60/2 = 30$ marbles.

5. A clock chimes once at 1:00, twice at 2:00, three times at 3:00, and so on. Starting at 12:30, how many times will the clock chime in 6 hours?
   Answer: 21
   The clock will chime $1 + 2 + 3 + 4 + 5 + 6 = (1 + 6) + (2 + 5) + (3 + 4) = 3 * 7 = 21$ times.

6. Scott bought a record collection for \$10, sold it for \$15, bought it back for \$20, and finally sold it for \$25. How much money did Scott make or lose?
   Answer: 10 or \$10 or Made 10
   In total, Scott made $\$15 + \$25 = \$40$ for selling the record collection. However, he spent a total of $\$10 + \$20 = \$30$ buying the record collection. Thus, he made $\$40 - \$30 = \$10$ in the end.

7. The physical education department tells Veena during sophomore year that she is 5'2" tall. In junior year she is told that she is 62.25" tall. By how many inches did Veena grow?
   Answer: $\frac{1}{4}$ or 0.25
   Supposedly, Veena grew $62.25 - 62 = 0.25$ inches.

8. The Bergen County Academies' field measures 13 meters by 17 meters. What is the total area?
   Answer: 221
   The field is $13 * 17$ square meters. Area = length $*$ width $= 13 * 17 = 221$ square meters.

9. Evaluate the following: $429863 + 228987 + 536741$
Answer: 1195591
$429863 + 228987 + 536741 = 1195591$

10. Rachel likes to collect gemstones. Her favorite gems are peridots. Out of her 147 gems, 74 are peridots. How many gems does she have that are not peridots?
Answer: 73
Rachel has $147 - 74 = 73$ gemstones that are not peridots.

11. Mark writes three math questions Monday, five math questions on Tuesday, seven Math questions on Wednesday, and so on. How many questions will he have written, in total, at the end of Sunday?
Answer: 63
He would have written $3 + 5 + 7 + 9 + 11 + 13 + 15 = 7 * 9 = 63$ questions.

12. If Jordan eats three pieces of candy every two days and Elen eats four pieces of candy every three days, how many pieces of candy will both of them have eaten in twelve days?
Answer: 34
Jordan will have had $12\text{days} * \frac{3\text{pieces}}{2\text{days}} = 18$ pieces and Elen will have had $12\text{days} * \frac{4\text{pieces}}{3\text{days}} = 16$ pieces of candy, so both of them will have had $18 + 16 = 34$ pieces of candy in twelve days.

13. Hyesoo accidentally subtracted 5 instead of dividing by 5 on a math problem. Her answer was 15. What should her answer have been?
Answer: 4
Work backwards to find the answer. Since she subtracted 5, we can reverse that by adding 5 to get the original number. Then we can divide that number by 5 to get the right answer. $15 + 5 = 20$, $\frac{20}{5} = 4$

14. How many hours are there in $3\frac{1}{4}$ days?
Answer: 78
There are 24 hours in one day, which means that there are $24 * 3 = 72$ hours in 3 days. In $\frac{1}{4}$ of a day, there are $24 * \frac{1}{4} = 6$ hours. Total, there are $72 + 6 = 78$ hours in $3\frac{1}{4}$ days.

15. Sylvia has 5 apples and 3 bananas. Each apple costs a dollar and each banana costs a half dollar. How many *cents* did she spend to buy all of them?
Answer: 650
Since there are 100 cents in one dollar, Sylvia spent $(5 * 100) + (3 * 50) = 650$ cents.

16. Every time Ethan enters Mr. Holbrook's room, he drinks three cans of soda. If Ethan enters Mr. Holbrook's room three times a day, how many cans of soda will he have had after five days?
Answer: 45
Ethan drinks three cans of soda each time he enters Mr. Holbrook's room. Since he enters Mr. Holbrook's room three times a day, he drinks $3 * 3 = 9$ cans of soda per day, which means that he drinks $9 * 5 = 45$ cans of soda every five days.

17. What is the perimeter of a square of area 16?
Answer: 16
If a square has an area of 16, that means that it has a side length of $\sqrt{16} = 4$. Thus the perimeter of the square is $4 + 4 + 4 + 4 = 4 * 4 = 16$

18. What is the area of a triangle with base 7 and height 4?
Answer: 14

The formula for the area of a triangle is $a = \frac{1}{2}bh$, where b is the length of the base and h is the height of the triangle. Thus the area of the triangle in question is $(\frac{1}{2})(7)(4) = 14$

19. Find the sum of the numbers from 1 to 15 inclusive.
Answer: 120
$1 + 2 + 3 + ... + 14 + 15 = (1 + 15) + (2 + 14) + (3 + 13) + ...(7 + 9) + 8 = (7 * 16) + 8 = 120$

20. Sang has to collect and dry wildflowers for his biology project. He finds 8 each time he goes out, but the first 3 always rot before he can get home to dry them. How many times does he have to go out to get the minimum requirement of 25?
Answer: 5
He collects $8 - 3 = 5$ good ones each time, so he must go 5 times to get $5 * 5 = 25$ flowers.

21. It takes 20 of Beowulf's men to beat a dragon in battle. If Beowulf has 170 men, how many dragons could they beat in battle?
Answer: 8
Since it takes 20 of Beowulf's men to defeat one dragon, Beowulf's men can handle $170/20 = 8.5$ dragons in a battle. Half-dragons do not count, so Beowulf's men can handle 8 dragons in battle.

22. In the ARML song contest, Ben and Joe wanted to sing a duet. However, their timing was terrible: Ben sang at 60 beats per minute and Joe sang at 80 beats per minute. If they sang their notes on the first beat together, how many beats will they be apart at the end of the song, 2 minutes and 45 seconds later?
Answer: 55
2 minutes and 45 seconds $= 2\frac{3}{4}$ minutes $= 11/4$ minutes. In this time, Ben will sing $11/4 * 60 = 165$ beats and Joe will sing $11/4 * 80 = 220$ beats. The difference is $220 - 165 = 55$ beats.

23. How many integers equal their own squares?
Answer: 2
The integers that equal their own squares are 0 and 1.

24. Before the big organic chemistry test, the last of 6 in the year, Arthur realizes that he needs an 88 to get a 93 average. What is his average so far?
Answer: 94
A 93 average means that the sum of the scores of all of his tests so far is $93 * 6 = 558 - 88 = 470$, which means that his average is $\frac{470}{5} = 94$

25. An adult working alone requires three hours to do a certain job. A child working alone requires six hours to do the same job. How many hours will it take the adult and child, working together, to do this job?
Answer: 2
The adult can finish $\frac{1}{3}$ of the job per hour. The child can finish $\frac{1}{6}$ of the job per hour. Together, they can finish $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$ of the job per hour. Thus, it would take them 2 hours to finish.

26. At the Bergen County Academies, a *club* consists of 10 students. A subcommittee must be formed by choosing 3 students from this club. How many possible subcommittees can be formed?
Answer: 120
$\binom{10}{3} = \frac{10!}{3!7!} = \frac{10*9*8}{3*2*1} = \frac{720}{6} = 120$

27. At age 8, Christine decided to start saving money from her allowance. She saved \$2 a month the first year, \$3 a month the second year, \$4 a month the third year, etc. She turned 18 today. How much money has Christine saved so far?

Answer: 780
Christine has $(2*12)+(3*12)+(4*12)+...+(10*12)+(11*12) = 12(2+3+...+9+10+11) = 12*65 = \$780$ saved away.

28. What is the number halfway between $\frac{1}{11}$ and $\frac{1}{7}$?
Answer: $\frac{9}{77}$
Find the arithmetic mean of the two fractions. Add the two fractions: $\frac{1}{11}+\frac{1}{7} = \frac{7+11}{11*7} = \frac{18}{77}$. Divide this sum by 2 to yield $\frac{9}{77}$.

29. Kevin, Watson, Robert, and Ricky are all standing in a line. Robert is standing somewhere between Kevin and Watson. Ricky is standing immediately to the right of Watson, but is not standing next to Robert. Who is standing next to the person all the way on the right?
Answer: Watson
From the first clue, the configuration of Robert, Kevin and Watson is either KRW or WRK. From the next clue, we know that the order cannot be WRK; if it were, then if Ricky were standing immediately to the right of Watson, then he would be standing next to Robert. However, this is not true, so the order must be KRW. Then Ricky is standing immediately to the right of Watson. So the order of people is Kevin, Robert, Watson, Ricky. The person standing next to the person all the way on the right (Ricky) is Watson.

30. Robert has two watches, one which loses 6 seconds every 24 hours and one which gains 1 second per hour. He sets both of them to the correct time at 6 : 00 p.m. How many hours will pass before the difference between the time shown on both watches is 1 minute?
Answer: 48 or 48 hours
After 24 hours, the difference will be $6 + 24 * 1 = 30$ seconds. After $2 * 24 = 48$ hours, the difference will be $2 * 30 = 60$ seconds, or one minute.

31. When a number is divided by 16, the remainder is 5. What is the remainder when the same number is divided by 4?
Answer: 1
We can describe the number as an expression. Say the number is $16x + 5$. This is the same as $16x + 4 + 1$. $16x + 4$ is divisible by 4, so the remainder when $16x + 4 + 1$ is divided by 4 is 1. Alternatively, you can try a test case, such as 21 (since when 21 is divided by 16, the remainder is 5). When 21 is divided by 4, the remainder is 1.

32. Find the largest factor of $111,111$ less than $111,111$.
Answer: 37037
If d is the smallest factor, $111111/d$ must be the largest factor. We look for the smallest factor: it is not even, so 2 is not a divisor. The sum of all the digits of $111,111$ is $1+1+1+1+1+1 = 6$ so it is divisible by 3. $111,111/3 = 37037$ is the largest factor.

33. Two passenger trains traveling in opposite directions meet and pass each other. Each train is 1/24 miles long and is traveling at 50 miles per hour. How many seconds after the front parts of the trains meet will their rear parts pass each other?
Answer: 3 or 3 seconds
The distance traveled is just the length of the train, which is 1/24 miles. Since the train is traveling at 50 miles per hour, (50 miles per hour)(time) = 1/24 miles, which means that the time is 1/1200 hours = 3 seconds .

34. The chickens, ducks and pigs in Farmer Lee's barn have the same number of heads and have a total of 72 legs. How many pigs are in the barn?

Answer: 9
Let c be the number of chickens, d the number of ducks and p the number of pigs. Then $c = d = p$ and $2c + 2d + 4p = 72 = 2p + 2p + 4p = 8p$. Hence $p = 72/8 = 9$.

35. A rectangle is 12 m wide and 5 m long. If the width is reduced by 10% and the length is increased by 20%, what is the percentage change in the area (from old to new)?
Answer: 108% or 1.08 or 8% more
$0.9 * 1.2 = 1.08$

36. Find the sum of the counting numbers from 1 to 25 inclusive.
Answer: 325
Notice that $1 + 25 = 26, 2 + 24 = 26, 3 + 23 = 26$, and so on up until $12 + 14 = 26$. There are 12 pairs of numbers that add to 26, with 13 left over. Thus, the sum is $12 \times 26 + 13 = 325$.

37. If the sum of 5 even numbers in a row is 320, what is the smallest of the five even numbers?
Answer: 60
Let the five even numbers be $x, x + 2, x + 4, x + 6$, and $x + 8$. Then $5x + 20 = 320 \rightarrow 5x = 300 \rightarrow x = 60$.

38. Let $x = 99887766554433221100$. Find the remainder when x is divided by 9.
Answer: 0
The divisibility rule for 9 states that if the sum of the digits of a number is divisible by 9, then the number is divisible by 9. The sum of the digits of 99887766554433221100 is $9+9+8+8+7+7+6+6+5+5+4+4+3+3+2+2+1+1+0+0 = 90$, which is clearly divisible by 9. Thus, 99887766554433221100 is divisible by 9, and the remainder when it is divided by 9 is 0.

39. Edward's Bike Shop has a total of 32 bicycles and tricycles for rent. He checks all 74 wheels at the beginning of the season. How many tricycles does Edward have?
Answer: 10
We know that bicycles have 2 wheels and tricycles have 3 wheels. There are a total of 74 wheels. Let x be the number of bicycles and let y be the number of tricycles. Hence we have $2x+3y = 74$ and $x + y = 32$. Solving this equation, we find that there are 22 bicycles and 10 tricycles.

40. The residents of Pinnatug greet each other by tugging each other's ears simultaneously. At a gathering, a total of 136 pairs of tugs took place. If each person tugged every other person's ear exactly once, how many residents attended?
Answer: 17
Let x be the number of residents of Pinnatug. Each resident tugs $x - 1$ residents' ears. Hence $x(x-1)$ will be the number that each resident tugs another's ear. However this also doubles the number. So the expression becomes $\frac{x(x-1)}{2} = 136$, which gives $x = 17$ as the positive solution.

41. How many factors does the number 3300 have?
Answer: 36
The prime factorization of 3300 is $2^2 * 3 * 5^2 * 11$. To make a factor of 3300, there are three choices for the power of $2 : 0, 1$, and 2. Similarly, there are two choices for the power of $3[0, 1]$, three choices for $5 : [0, 1, 2]$, and two choices for the power of $11 : [0, 1]$. To find the total number of factors of 3300, we multiply these numbers: $3 * 2 * 3 * 2 = 36$.

42. The mean of a set of 5 numbers is 32. The number 132 is removed from the set. By how much is the mean reduced?
Answer: 25
Let y be the sum of the 4 remaining elements, then $(y+132)/5 = 32$, hence $y = 5*32-132 = 28$. Therefore $y/4 = 28/4 = 7$. The answer is $32 - 7 = 25$.

43. The biology flower picking project is over, and the students get their grades. Their teacher gives 4 points for a correct flower identification and -2 points [takes 2 points off] for an incorrect one. If Sang collected 25 flowers and has a grade of 70, how many flowers did he correctly identify?
Answer: 20
Set up a system of equations: $x + y = 25$, $4x - 2y = 70$. x =correct, y =incorrect. $2(x + y) = 2(25) = 2x + 2y = 50$. $6x = 120$. $x = 20$, $y = 5$. 20 correct flowers.

44. Sandhya numbers the pages in her 3-subject notebook from 1 to 132. How many 1's did she use in the page numbers?
Answer: 67
There are 14 times that the number 1 is used in the one's place $(1, 11, 21, \ldots, 131)$. There are 20 times when the number 1 appears in the tens place $(10 - 19$ and $110 - 119)$. There are 33 times that the number 1 appears in the hundreds place $(100-132)$. Thus, the number 1 appears $14 + 20 + 33 = 67$ times.

45. David borrowed 3/5 of Mark's money and spent 3/5 of it on music. He then returned the remaining money to Mark, which was 48 dollars less than the amount Mark had after lending him the money. How many dollars did Mark have originally?
Answer: 300
Let x be the number of dollars Mark had originally. We can write an equation to describe this situation: $(3/5)x - (3/5)^2x = x - (3/5)x - 48$. Solving this gives $x = 300$

46. What is the smallest integer that $864,000$ must be multiplied by to get 6 terminating zeros?
Answer: 125
The prime factorization of $864,000$ is $2^8 * 3^3 * 5^3$. To get 6 terminating zeros, we need 6 pairs of 2's and 5's. Since we already have at least 6 2's and 3 5's, we only need three more fives to get 6 terminating zeros. Thus, the smallest integer that $864,000$ can be multiplied to get 6 terminating zeros is $5 * 5 * 5 = 125$.

47. Sherry packed 480 math books in boxes so that the number of boxes was 16 less than twice the number of math books in each box. How many boxes did Sherry use?
Answer: 24
Let b be the number of boxes and p be the number of math books in each box. We can write a system of equations for this situation: $b = 2p - 16$ and $b * p = 480$ Combining these two equations, we get: $b(b + 16)/2 = 480$. Solving for b, we find that $b = 24$.

48. When Ben has a "sharpie battle," he has a 1/3 chance of poking his opponent's arm and a 1/5 chance of poking their neck (neither affects the other). When he faces Yoonjoo, he gets three chances to poke her. What is the probability that he pokes her arm, her neck, and her arm again in that order?
Answer: 1/45
They are independent events, so $P(A \text{ and } B) = P(A) * P(B)$ applies. $P = 1/3 * 1/5 * 1/3 = 1/45$.

49. Andy flips a coin 7 times. What is the probability that all 7 flips are heads?
Answer: $\frac{1}{128}$
The probability for a coin to land on heads is $\frac{1}{2}$. Therefore, for each flip, there is a 1/2 chance of getting heads. The coin is flipped 7 times, which gives $\frac{1}{2} * \frac{1}{2} * \frac{1}{2} * \frac{1}{2} * \frac{1}{2} * \frac{1}{2} * \frac{1}{2} = \frac{1}{2^7} = \frac{1}{128}$.

50. $\log_2 32 = ?$
Answer: 5
$x = \log_2 32 \rightarrow 2^x = 32 \rightarrow x = 5$.