# 5th Grade Competition

Bergen County Academies
21 October 2007

1. A student has to compile 250 questions for a math competition. She asked each student on the math team to write three questions. However, out of 125 students, only 15 students wrote questions. Two students wrote the same three questions. How many questions does the student now have to write to complete the competition?
   **Answer: 208**
   If 2 students wrote the same questions, then only one set could be included, which in effect means 14 students wrote questions. Since each student wrote 3 questions, she received $3 * 14 = 42$ questions. Thus she needs to write $250 - 42 = 208$ questions.

2. Find $8 + 10 \div 2 + 4 * 2 - 21 \div 7$.
   **Answer: 18**
   Multiplications and divisions occur first. Hence, $8 + 10 \div 2 + 4 * 2 - 21 \div 7 = 8 + (10 \div 2) + (4 * 2) - (21 \div 7) = 8 + 5 + 8 - 3 = 18$.

3. $\frac{3}{5} \div \frac{1}{10} = ?$
   **Answer: 6**
   $\frac{3}{5} \div \frac{1}{10} = \frac{3}{5} * \frac{10}{1} = \frac{30}{5} = 6$

4. Ashley is 18 years old and Scott is twice her age. If Ashley's mom is 10 years older than Scott, how old is Ashley's mom?
   **Answer:** 46
   If Ashley is 18 years old and Scott is twice her age, then Scott is $18 * 2 = 36$ years old. Thus if Ashley's mom is 10 years older than Scott, she is $36 + 10 = 46$ years old.

5. Veena began reading Harry Potter and the Deathly Hallows at 12:24AM. If she reads at 253 pages per hour and there are 759 pages, at what time did she finish reading?
   **Answer: 3:24 AM**
   It took her $\frac{759}{253} = 3$ hours to read the book. 3 hours after 12:24AM is 3:24AM.

6. For every 250 words Lorenzo types, he makes 4 mistakes. If he writes a 5000 word literature paper, how many mistakes can he be expected to make?
   **Answer: 80**
   $\frac{4}{250} = \frac{x}{5000} \rightarrow 250x = 20,000 \rightarrow x = 80$

7. $\frac{1}{5} + \frac{1}{15} = ?$
   **Answer: 4/15**
   $\frac{1}{5} + \frac{1}{15} = \frac{3}{15} + \frac{1}{15} = \frac{4}{15}$.

8. If one circle's diameter is another circle's radius, then the smaller circle's area is what percentage of the larger circle's area?
   **Answer: 25 or 25%**
   Substitute numbers - let $r = 2$ be the radius of the larger circle. Then the area of the larger

circle is $A = \pi r^2 = \pi(2)^2 = 4\pi$. The radius of the smaller circle is r=1, so the area is $\pi$. To find the percentage, we take the ratio of the two areas: $\pi/4\pi = 1/4$, or 25%.

9. Suppose that it takes Chirag on average 45 minutes to paint a fence. It takes Trent on average 30 minutes to paint the same fence. If Chirag and Trent worked together to paint the fence, how many minutes would it take them to finish?
**Answer: 18**
Let the total time it takes Chirag and Trent to paint the fence be $t$. In one minute, Chirag can paint 1/45 of the fence and Trent can paint 1/30 of the fence. Then in $t$ minutes, Chirag paints $t/45$ of the fence and Trent paints $t/30$ of the fence; together they paint $t/45 + t/30$ of the fence in $t$ minutes. Since we want them to finish, $t/45 + t/30 = 1$, which gives $t = 18$ minutes.

10. It takes Carrie 40 minutes to walk between her home and her school. One morning she walked half way to school and remembered that she had left her calculator at home. She ran home. It took 5 minutes to find her calculator when she got home. Then she ran all the way to school. She runs twice as fast as she walks. How many minutes more than usual did it take for her to get to school?
**Answer: 15**
$40 \times \frac{1}{2} + 20 \times \frac{1}{2} + 5 + 20 - 40 = 15$.

11. Find $|4 - 7| + |7 - 4|$
**Answer: 6**
Notice that $|4 - 7| = |-3| = 3$ and $|7 - 4| = |3| = 3$ then $|4 - 7| + |7 - 4| = 3 + 3 = 6$

12. Scott bought a record collection for \$10, sold it for \$15, bought it back for \$20, and finally sold it for \$25. How much money did Scott make or lose?
**Answer: 10 or \$10 or Made 10**
In total, Scott made $\$15 + \$25 = \$40$ for selling the record collection. However, he spent a total of $\$10 + \$20 = \$30$ buying the record collection. Thus, he made $\$40 - \$30 = \$10$ in the end.

13. The houses on Gauss Street are numbered consecutively from 1 to 602. How many brass digits are needed to form all the house numbers?
**Answer: 1698**
There are 9 one digit numbers ($1 \approx 9$), 90 two digit numbers ($10 \approx 99$), and $(602-100+1) = 503$ three digit numbers ($100 \approx 602$). Thus $9 * 1 + 90 * 2 + 503 * 3 = 1698$ brass digits are needed.

14. What is the 10th term in the arithmetic sequence $1, 5, 9, \ldots$?
**Answer: 37**
The first term is $1 * 4 - 3$, the second is $2 * 4 - 3$, the third is $3 * 4 - 3$, and so on. Thus, the tenth term will be $10 * 4 - 3 = 37$.

15. How many degrees are in the measure of the smaller angle that is formed by the hands of a clock when it is 3:00?
**Answer: 90**
Each hour amounts to $360/12 = 30$ degrees, so 3 hours will give rise to an angle of $30 * 3 = 90$.

16. Find the 6th term in the geometric sequence $3, 6, 12, \cdots$
**Answer: 96**
The ratio is 2, so we have to multiply each term by 2 to find the next one. So, the fourth term is $12 * 2 = 24$, the fifth term is $24 * 2 = 48$, and the sixth term is $48 * 2 = 96$.

17. $\frac{3+6+9+12+...+291+294}{4+8+12+16+...+388+392} =?$
**Answer: 3/4**
Factor the numerator and denominator to get $\frac{3(1+2+3+...+97+98)}{4(1+2+3+...+97+98)} = 3/4$

18. How many integers equal their own squares?
**Answer: 2**
The integers that equal their own squares are 0 and 1.

19. A bag of marbles can be completely divided in equal shares among 2, 3, 4, 5, or 6 friends. What is the least number of marbles that the bag could contain?
**Answer: 60**
The least common multiple of 2, 3, 4, 5, and 6 is $2^2 * 3 * 5 = 60$.

20. Find the sum of the counting numbers from 1 to 25, inclusive.
**Answer: 325**
Notice that $1 + 25 = 26, 2 + 24 = 26, 3 + 23 = 26$, and so on up until $12 + 14 = 26$. There are 12 pairs of numbers that add to 26, with 13 left over. Thus, the sum is $12 \times 26 + 13 = 325$.

21. Andrew has a new 3-ft-wide bookcase with two shelves, each 15 in. high. He plans to store his CDs on the shelves using new CD racks. Each CD rack is 17 in. wide and 7 in. high, and holds 3 stacks of 12 CDs. How many CDs will he be able to store?
**Answer: 288**
Each CD rack holds 36 CDs. (12 * 3 = 36) Each shelf is 3 ft or 36 in. wide, and therefore wide enough to hold 2 CD racks.(2 * 17 = 34)Each shelf is 15 in. high, and therefore high enough to hold 2 layers of CD racks(2 * 7 = 14) Therefore, each shelf holds 4 CD racks, or 36 * 4 = 144 CDs.There are two shelves, each holding 144 CDs, or 288 CDs in all.

22. If the sum of 5 consecutive even numbers is 320, what is the smallest of the five even numbers?
**Answer: 60**
Let the five even numbers be $x$, $x + 2$, $x + 4$, $x + 6$, and $x + 8$. Then $5x + 20 = 320 \rightarrow 5x = 300 \rightarrow x = 60$.

23. How many factors does the number 3300 have, including 1 and 3300?
**Answer: 36**
The prime factorization of 3300 is $2^2 * 3 * 5^2 * 11$. To make a factor of 3300, there are three choices for the power of $2 : 0, 1$, and 2. Similarly, there are two choices for the power of $3 : 0$ and 1. There are three choices for the power of $5 : 0, 1$, and 2. There are two choices for the power of 11: 0 and 1. To find the total number of factors of 3300, we multiply these numbers. So, 3300 has $3 * 2 * 3 * 2 = 36$ factors.

24. The mean of a set of 5 numbers is 32. The number 132 is removed from the set. By how much is the mean reduced?
**Answer: 25**
Let $y$ be the sum of the 4 remaining elements, then $(y+132)/5 = 32$, hence $y = 5*32-132 = 28$. Therefore $y/4 = 28/4 = 7$. The answer is $32 - 7 = 25$.

25. The circumference of a circle is $22\pi$. If its radius is halved, then what is the area of the resulting circle (in terms of $\pi$)?
**Answer:** $\frac{121\pi}{4}$
$C = \pi d$. Therefore the diameter is 22 and the radius is 11. $A = \pi r^2$, so the area of a circle with radius of $\frac{11}{2}$ is $\frac{121\pi}{4}$.

26. Robert has two watches, one which loses 6 seconds every 24 hours and one which gains 1 second per hour. He sets both of them to the correct time at 6 : 00 p.m. How many hours will pass before the positive difference between the time shown on both watches is 1 minute?
**Answer: 48 or 48 hours**
After 12 hours, the difference will be 15 seconds. After $4 * 12 = 48$ hours, the difference will be $4 * 15 = 60$ seconds, or one minute.

27. At the HMMT math competition sophomore year, Rachel decided to play a trick on Ethan by putting salt in his soda. She put 40 grams of salt into his 12 oz glass of soda. However, he saw her do this, so when she stepped away, he poured 3 oz of his soda into her glass which had contained only 7 oz. When she came back and took a 1.5 oz gulp, how many grams of salt did she drink?
**Answer: 1.5 or 3/2**
40 grams / 12 oz * 3 oz transferred = 10 grams of salt transferred into Rachel's glass. Rachel's glass now contains 10 grams of salt and $3 + 7 = 10$ oz soda. 10 grams / 10 oz * 1.5 oz drink = 1.5 grams of salt.

28. A florist buys roses at \$0.50 apiece and sells them for \$1.00 apiece. If there are no other expenses, how many roses must be sold in order to make a profit of \$300?
**Answer: 600**
Each rose brings a profit of \$1.00 minus \$0.50, or \$0.50. The number of roses that need to be sold to make a profit of \$300 is \$300 / \$0.50 = 600 roses.

29. $\frac{1}{2} * \frac{2}{3} * \frac{3}{4} = ?$
**Answer:** $\frac{1}{4}$
The 2's and 3's cancel to yield $\frac{1}{2} * \frac{2}{3} * \frac{3}{4} = \frac{1}{4}$

30. $\frac{7^2+7}{7} = ?$
**Answer: 8**
$\frac{7^2+7}{7} = \frac{49+7}{7} = \frac{56}{7} = 8$

31. A student at the Academies has to be at school by 8:00 a.m. It takes him 15 minutes to get dressed, 20 minutes to eat and 35 minutes to get to school. What time should he get up?
**Answer: 6:50 AM**
9 hours $-(15 + 20 + 35) = 9$ hours $-70$ minutes $= 7$ hours $+120$ minutes $-70$ minutes $= 7$ hours and 50 minutes $= 6{:}50AM$

32. At a party, each of 8 friends was given a game card. To play the game, each person had to walk around and trade a card with every other person at the party. How many trades took place?
**Answer: 28**
2 people made 1 trade, 3 people made 3 trades, 4 people made 6 trades, and so on. 8 people made 28 trades. Otherwise, $\binom{8}{2} = 28$

33. I am a four-digit number with no two digits the same. My ones digit is twice my thousands digit and one less than my tens digit. My hundreds digit is the difference between my tens and my thousands digit. My thousands digit is an odd number less than 6. What number am I?
**Answer: 3476**
You can start with 5, 3, and 1 for the thousands digit. Eliminate 5, since you cant double it to get the ones digit. Now you have 3 and 1 with the possibilities 3__6 or 1__2.

Since the ones digit is one less than the tens digit, you can have 3_76 or 1_32.

Finally, the hundreds digit is the difference between the tens digit and the thousands digit, so the number is 3476 or 1232. But no digits can be repeated, so the number must be 3476.

34. $27^{2/3} = ?$
**Answer: 9**
$27^{2/3} = 27^{(1/3)(2)} = 3^2 = 9$

35. Sujin opened her math book and found that the sum of the facing pages was 243. What was the larger page of the two pages where she opened the book?
**Answer: 122**
The average of the two pages is $\frac{243}{2} = 121.5$, which means that the two consecutive pages are 121 and 122. Hence the larger page is 122.

36. A gasoline tank on a certain tractor holds 16 gallons of gasoline. If the tractor requires 7 gallons to plow 3 acres, how many acres can the tractor plow with a tankful of gasoline?
**Answer: $\frac{48}{7}$ or $6\frac{6}{7}$**
Let $x$ be the number of acres the tractor can plow with 16 gallons. You can set up a ratio of the number of gallons of gasoline used to the number of acres plowed. Therefore, you have $\frac{7}{3} = \frac{16}{x}$. Cross-multiplying and solving for $x$ gives you the answer of $x = \frac{48}{7}$ acres, or $6\frac{6}{7}$ acres.

37. The volume of a cube with edge length of 2 meters is what fraction of the volume of a cube with edge length of 4 meters?
**Answer: $\frac{1}{8}$**
The volume of the first cube $v_1 = 2^3 = 8$. The volume of the second cube $v_2 = 4^3 = 64$. $\frac{v_1}{v_2} = \frac{8}{64} = \frac{1}{8}$

38. Dr. Abramson's AP Calculus class holds a contest to guess the number of candies in the jar. Three people have already guessed, and their guesses are 315, 350, and 327. One of the guesses is off by 26, one is off by 14, and one is off by 9. How many candies are in the jar?
**Answer: 341**
The difference between the highest and lowest guess is 35, so one of these guesses can be off by 26. Suppose 350 is off by 26. Then there would be 324 candies in the jar. 315 is 9 away from 324, but 327 is only 3 away from 324. So the conditions are not met. Now try 315. If 315 is off by 26, there are 341 candies in the jar. This is 9 away from 350, and 14 away from 327. So 341 is the correct number of candies.

39. When Ben has a "sharpie battle," he has a 1/3 chance of poking his opponent's arm and a 1/5 chance of poking their neck (neither affects the other). When he faces Yoonjoo, what is the probability that he pokes her arm, her neck, and her arm again in that order?
**Answer: 1/45**
They are independent events, so $P(A \text{ and } B) = P(A) * P(B)$ applies. $P = 1/3 * 1/5 * 1/3 = 1/45$.

40. Mr. Teacher is buying 2-liter bottles of soda for his end of the year class party. A 2-liter bottle contains 8 servings. How many bottles must he buy for 20 students, 2 parents, and himself?
**Answer: 3**
Mr. Teacher needs to serve $20 + 2 + 1 = 23$ people. Thus he needs $2 \leq 23/8 \leq 3$ bottles of soda. He needs 3 bottles to serve everyone.

41. Ernie has 10 baseball cards. He buys 6 more cards at the store. Ernie then gives Bert 5 cards. How many baseball cards does Ernie now have?
**Answer: 11**
Ernie buys 6 cards, which means that he has $10 + 6 = 16$ cards. Then he gave Bert 5 cards, which means that he has $16 - 5 = 11$ cards.

42. Solve for $x : 5 + 47x = 23 - 43x$
**Answer: 1/5 or 0.2**
First, add $43x$ to both sides to get $5+90x = 23$. Then subtract 5 from both sides to get $90x = 18$. Finally, divide both sides by 90. After using algebra to simplify, $x = 18/90 = 1/5 = 0.2$.

43. Ian has a container holding 134 quarts of mixtures of 30% NaCl and 70% $H_2O$. Ian has a second container holding 50 quarts of mixtures of 50% NaCl and 50% $H_2O$. If he mixes them, what percent, to the nearest whole percent, the mixture will be NaCl?
**Answer: 35**
$100 * \frac{134*(\frac{30}{100})+50*(\frac{50}{100})}{(134+50)} = 35$ to the nearest whole percent.

44. Find the only four-digit number that is divisible by 25, 27, and 8.
**Answer: 5400**
The prime factorizations of these three numbers are $5^2$, $3^3$, and $2^3$. Their least common multiple is $2^3 * 3^3 * 5^2 = 5400$. Any multiple of 5400 will be more than four digits, so 5400 is the only four-digit number divisible by $25, 27$ and 8.

45. What is the surface area of a cube with side length 1.5?
**Answer: 13.5**
The surface area of a cube is equal to 6 times the area of a single side because all 6 sides have the same area. The area of a single side is $1.5 * 1.5 = 2.25$. Hence the surface area is $2.25 * 6 = 13.5$.

46. The chickens, ducks and pigs in Farmer Lee's barn have the same number of heads and have a total of 72 legs. How many pigs are in the barn?
**Answer: 9**
Let c be the number of chickens, d the number of ducks and p the number of pigs. Then c=d=p and 2 c + 2 d + 4p = 72= 2 p + 2 p + 4 p = 8 p. Hence p = 72/8 = 9.

47. $(((x)^6)^{1/3}) + (((2x)^3)^{1/3}) + (1)^{1/3} = 0$. $x$ =?
**Answer: -1**
$(((x)^6)^{1/3}) + (((2x)^3)^{1/3}) + (1)^{1/3} = x^2 + 2x + 1 = 0 \rightarrow (x + 1)(x + 1) = 0 \rightarrow x = -1$.

48. Two passenger trains traveling in opposite directions meet and pass each other. Each train is 1/24 miles long and is traveling at 50 miles per hour. How many seconds after the front parts of the trains meet will their rear parts pass each other?
**Answer: 3**
The distance traveled by each train while the trains pass each other is the length of the train, which is 1/24 miles. Use (rate)(time)=distance: (50 miles per hour)(time) = 1/24 miles, which means that the time is 1/1200 hours. Since there are $60 \times 60 = 3600$ seconds per hour, 1/1200 hours is equal to (1/1200 hours)(3600 seconds/hour) = 3 seconds.

49. Blocks of molding clay are 9 inches by 6 inches by 3 inches. How many whole blocks are needed to mold a cylindrical sculpture 13 inches high and 6 inches in diameter? Use 3.14 as an approximation of Pi.
**Answer: 3**
$v_1$=volume of a block $= 9 * 6 * 3 = 162$. $v_2$=volume of a sculpture $= 3.14 * (3^2) * 13 = 367.38$. $v_2/v_1 = 2.26778$. Hence the answer is 3.

50. If $x = 9$, compute the value of $x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$.
**Answer: 1,000,000 ($10^6$)**
Notice that $x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1 = (x + 1)^6 = (9 + 1)^6 = 10^6 = 1000000$