5th Grade Competition

Bergen County Academies Math Competition
19 October 2008

1. Before taking the AIME, a student notices that he has a pack of Smarties, two bags of chips, and a Hershey's kiss on his desk. Each pack of Smarties has 50 calories, each bag of chips has 120 calories, and each Hershey's kiss has 10 calories. If the student eats all of the food on his desk during his test, then how many calories will he have eaten?

2. $12 + 34 - 56 - 78 + 90 =$?

3. Pavel is driving at 75 miles per hour. Looking into his rearview mirror, he notices that he is traveling 17 miles per hour faster than the truck behind him. At what speed is the truck traveling, in miles per hour?

4. Nikhil says the word 'horse' ten times per hour. If he is awake for 16 hours per day, then how many times will he say the word 'horse' in a 24-hour period? (Nikhil does not talk in his sleep.)

5. Find the perimeter in inches of a square that has area 49 ft^2.

6. $(300 + 50 + 8) \times (1000 + 20 + 4) = $?

7. Right triangle ABC has a right angle at C and side lengths $\overline{AC} = 6$ and $\overline{BC} = 3$. Compute the area of $\triangle ABC$.

8. If a ball is moving with a constant speed of 5 meters per second over a duration of 24 seconds, how many meters does the ball move?

9. Find the largest integer value of a for which the statement "2048 is divisible by 2^a" is true.

10. Robert's backyard measures 37 by 53 feet. What is the area of Robert's backyard, in ft^2?

11. Nikhil is playing ping pong. In a game to 15, he was losing 10-4. Then, Nikhil scored a more points than his opponent and won the game. What is the smallest possible value of a?

12. Robert's Pokemon card collection contains 9135 cards, including 289 holographic ones. Robert gives away 568 of his non-holographic cards. How many non-holographic cards does he have now?

13. Tom has a rectangular field of dimensions 15 x 36 feet. A post must be placed no more than three feet from any other post, and each corner must have a post. What is the minimum number of posts that Tom needs to surround his field?

14. Compute the hypotenuse of a right triangle that has legs of lengths 3 and 9.

15. In how many ways can 9 be written as a sum of three different positive integers?

16. The area of $\triangle ABC$ is 35. If $\overline{BC} = 10$, then find the height from A to $\overline{BC}$.

17. How many ways can Ally, Brett, and Candy be lined up so that Ally is standing next to her best friend Candy?

18. If it takes Alex two seconds to type a 5-letter word, then how many seconds will it take him to type 317 5-letter words? Ignore the time it takes to type punctuation and spaces.

19. In an auditorium, there are 6 rows of lights. Each row has 7 lights in it. If every light switch in the auditorium

controls exactly three lights, then how many light switches are in the auditorium?

20. The dimensions of a pool are 30 feet wide by 100 feet long by 8 feet deep. If water flows in to the pool at a rate of 150 cubic feet per minute, then how many minutes will it take for the pool to be full with water?

21. What is the area of a square with a diagonal of length 8?

22. Ian A. practices the French horn. If he practices 30 minutes per day from Monday to Friday and 45 minutes per day on Saturday and Sunday, then how many hours does he take to practice the French horn in 2 weeks?

23. The surface area of a soccer ball is two square feet. If 20% of the soccer ball is black, then what is the surface area, in square feet, of the white area on the ball?

24. In a sequence on integers, the first term is 1, the second and third terms are 2, the fourth, fifth, and sixth terms are 3, the next four terms are 4, and so on. What is the sum of the first 10 terms?

25. What is the volume of a square pyramid with base side length 5 and height 36?

26. A group of people are trying to sit themselves in rows. If they sit in rows of two, there is one person left over. If they sit in rows of five, there are four people left over. What is the smallest possible number of people in this group?

27. At math team, 5 bags of chips are equal to 2 cans of soda, and 7 boxes of Nerds are equal to 3 bags of chips. How many cans of soda are 105 boxes of Nerds equal to?

28. Find the area of trapezoid $ABCD$ given $\overline{AB} = 9$, $\overline{CD} = 7$, and the distance between parallel line segments $\overline{AB}$ and $\overline{CD}$ is 3.

29. If Marina can correctly differentiate between Ben and Joe only $\frac{4}{5}$ of the time, what are the chances that she correctly identifies Joe twice in a row?

30. If $1 - 7x = 2x$, then what is x?

31. Given that $n! = n(n-1)(n-2)\ldots 3 \cdot 2 \cdot 1$, compute $\dfrac{7! + 6!}{5!}$.

32. A trapezoid has bases with lengths 6 and 9, and has area 105. What is the height of the trapezoid?

33. What is the distance from the point $(3, 2)$ to the point $(0, 6)$?

34. Find the number of factors of 36.

35. How many zeroes does the number $2^7 \times 5^4 \times 11$ end in?

36. $\dfrac{(27^3)^{\frac{5}{9}}}{27} = ?$

37. $f(x)$ is a function such that $f(1) = 1$ and $f(2n + 1) = f(n) + 1$. For example, $f(3) = f(2 \cdot 1 + 1) = f(1) + 1 = 2$. Compute $f(127)$.

38. What is the degree measure of each angle in a regular hexagon?

39. For what value(s) of x does $\dfrac{x^2 - 4}{x - 2} = 0$?

40. Three cups of a mixed drink are made by mixing two cups of cranberry juice with one cup of carbonated water. To this mixture, one more cup of cranberry juice and one more cup of carbonated water are added. What percentage of the drink is now carbonated water?

41. How many integers less than 50 have exactly 2 distinct prime factors?

42. Nikhil is writing the page numbers in a book that has 102 pages. After a certain page number, Nikhil notices that he has written the same number of digits as he still has to write in order to finish numbering all 102 pages. What page number is this?

43. Jae's mom wants to buy a pair of pants that costs \$50. When she gets to the register, she finds out that the store is having a 10% discount. On top of this, she presents a 20% coupon to the cashier. How many dollars does Jae's mom end up paying for the pants?

44. On a football field, there are 15 blades of grass per square inch. If a football field measures 360 feet by 160 feet, then there are n blades of grass on the entire field. Find the leftmost three non-zero digits of n.

45. Compute $i^{2008} - i^{2006}$, given $i = \sqrt{-1}$.

46. Find two positive whole numbers a and b, both greater than 1, so that $ab - 1$ is the product of two consecutive positive integers and $a + b$ is as small as possible. Express your answer as the ordered pair (a, b) where $a < b$.

47. The incircle of $\triangle ABC$ touches $\overline{BC}$ at D, $\overline{AC}$ at E, and $\overline{AB}$ at F. Given $\overline{AF} = 3$, $\overline{BD} = 4$, and $\overline{CE} = 6$, find the perimeter of $\triangle ABC$.

48. Four farmers find 20 cows in a field and decide to split the cows between them (cows may not be split in half). Farmer A insists on having at least 1 more cow than farmer B, at least 2 more cows than farmer C, and at least 3 more cows than farmer D. What is the minimum number of cows Farmer A can take?

49. When Joyce goes out to buy clothes, she buys three shirts, each worth \$12, and a bathing suit worth \$24. If she leaves the store with $\frac{1}{3}$ of the money that she entered the store with, then how many dollars did Joyce enter the store with?

50. Find the sum of the first 16 terms of the arithmetic sequence 3, 7, 11, …