7th Annual Bergen County Academies Math Competition

Sixth Grade

Sunday, 18 October 2009

1 Rules

1. You may use space on your test paper and additional scrap paper to do work. Your answers must be written on the answer sheet. We will not look at answers written on your test paper.
2. Each problem has only one answer. If you put more than one answer for a problem, you will be marked wrong. When changing an answer, be sure to erase or cross out completely.
3. Write legibly. If the graders cannot read your answer, it will be marked incorrect.
4. Fractions should be written in lowest terms. For example, if the answer is $\frac{1}{2}$, then $\frac{2}{4}$ will not be accepted although the two fractions are numerically equal.
5. All other answers should be written in simplest form.
6. If a unit is indicated in the problem, the answer must be given in that unit. For instance, if the problem asks for the answer in hours, you cannot give your answer in minutes. Furthermore, you don't need to write the unit, as the graders will assume your answer is in the units asked for in the problem.
7. There is no penalty for guessing.
8. Ties will be broken based on the number of correct responses to the last ten questions. If a tie remains, then the correct responses to the last five questions will break the tie.
9. We will announce how much time is remaining often during the test.

2 Contest

1. $1 + 0 + 1 + 2 + 1 + 5 + 1 + 9 + 1 + 14 + 1 + 20 + 1 + 27 + 1 + 35 + 1 + 44 + 1 + 54 = ?$
2. Yi has to write an essay that is 1005 words long. He writes 10 words on the first day, 20 words on the second day, and 30 words on the third day. He has 10 days to finish the essay. Over the next seven days, what is the average number of words he will need to write per day in order to finish the essay?
3. A circle with circumference 12 has an inscribed square. Find the area of this square.
4. Kelly's favorite number is a positive integer less than 50. Both of the digits are odd. The tens digit is greater than the ones digit. What is Kelly's favorite number?
5. If the ratio of the surface area to the volume of a sphere is 1:3, what is its radius?
6. Let $f(x) = \frac{x^3-3x-9}{x^3-7x-6}$. Compute the sum of all distinct values of x for which $f(x) = 1$.
7. What is the largest prime factor of $34 \cdot 51 \cdot 68$?
8. There are fifty marbles in a jar. Three of them are white, and thirty-three of them are black. If I draw out a marble, what is the probability that it is neither white nor black?
9. Let $a \diamond b = \dfrac{a^2}{2} - b$. Evaluate $6 \diamond (4 \diamond 3)$.
10. If Pavel travels along the path from $(0, 0)$ to $(12, 16)$ in 5 seconds, what is his average speed?
11. If someone submits n problems for the Bergen County Academies math competition, they receive $\left\lfloor \dfrac{n}{12} + 1 \right\rfloor$ community service hours, where $\lfloor x \rfloor$ is the greatest integer less than or equal to x. What is the minimum number of problems a single person needs to submit in order to receive seven hours?
12. A cylinder with integer radius and height has surface area 42π. Find the minimum possible height of the cylinder.
13. In thirty years, Bob's age will have doubled. What is Bob's age now?
14. Mike is thinking of four *different* positive integers. Their product is 36. What is their sum?
15. Compute $i^{2008} - i^{2006}$, given $i = \sqrt{-1}$.
16. Rob swims two laps in a circular pool. He swims the first lap at a speed of 20 inches per second. He completes his second lap in 30 inches per second. What is his average speed of his swim, in inches per second?
17. How many integers from 1 to 1000 are multiples of 39 but not 13?
18. James was adding the numbers $1, 2, 3, \ldots$, and when he reached a certain number, the sum was 1,000. However, when he was checking his work, he founds out that he counted one of the numbers twice. What was that number?
19. A line with slope 1 is moved to the right 5 units, moved down 2 units, mirrored over the x-axis, then mirrored over the y-axis. What is the slope of the resulting line?
20. How many values of n are there such that $\dfrac{50}{n}$ is an integer?
21. If I roll two fair six-sided dice, what is the probability that the sum of the numbers on the tops is 8?

22. There are two colors of magic blobs: red blobs and blue blobs. Every minute, a red blob divides into two red blobs of equal volume and a blue blob divides into three blobs of equal volume: one red and two blue. If I start with a blue blob of volume 1, what volume of red blobs will I have in four minutes?

23. There are two numbers that satisfy the equation $(2x+3)(x-6)+(x-4)(2x+3)=0$. What is their sum?

24. At the academies, there is a game known as Mathathon. In Mathathon, points are scored by doing either "Picky Problems" or "Cute Questions." Each cute question is worth nine points and each picky problem is worth five points. Assuming there are an infinite number of both types of problems, and problems can only be scored in whole number amounts, how many scores are unattainable?

25. Hannah can build a house in twenty-four hours. Beth can build the same house in forty-eight hours. If Beth starts building the house and Hannah starts helping her a day later, in how many days will the house be built?

26. Let $a\%b$ be the remainder when a is divided by b. What is $(((((127\%64)\%32)\%16)\%8)\%4)\%2$?

27. The formula for converting Fahrenheit to Celsius temperatures is $C=\frac{5}{9}(F-32)$. Find the number of integer Fahrenheit temperatures between 0^o and 100^o that corresponds to an integer number of Celsius degrees.

28. The area of circle O is 4 square meters. If the diameter of O is doubled, what is the area of O in square meters?

29. In a Computer Science AB class, Watson argues with Pavel for 68% of the class time and with Dr. Nevard for 17% of the class time, but he never argues with both at the same time. The rest of the time, Watson does not argue with anyone. If Watson is not arguing with anyone for m% of the class, find m.

30. Jeff wants to order an appetizer and a drink at Boston Market. If Boston Market had one more appetizer, Jeff would have twenty more combinations to choose from. If Boston Market removed a drink, Jeff would have eight fewer choices. How many meals could Jeff choose from originally?

31. Kamran flips a quarter four times. What is the probability that he gets at least two heads?

32. Evaluate $\frac{1}{2}+\frac{1}{3}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{2^3}+\frac{1}{3^3}+...$

33. Find the area of the region in the xy-plane that satisfies $y>|x|$ and $y<5-|x|$.

34. There is a class of fourteen people. Suppose that the students are giving a presentation on math problems, so that on each of the first three days, four students present, and on the fourth day, two students present. If the day each student is assigned to present is chosen randomly, find the probability that Michael will present on the last day.

35. What is 70% of 70%? Express your answer as a percent.

36. Three fifth graders running at different speeds are racing around a circular tower. The fastest one can run around the tower in 5 minutes. The slowest one can run around the tower in 7 minutes. They start at the same point and they race around the tower in the same direction. After 35 minutes, they are all at the starting point. How long does it take for the second fastest runner to run around the tower?

37. $2+\left|1-\left|\frac{7}{4}\right|\right|=?$

38. A $3 \times 3 \times 3$ cube made up of twenty-seven $1 \times 1 \times 1$ cubelets has all of its faces painted red. The eight corner cubelets are removed. Find the difference in the surface areas of the cube before and after the cubelets were removed.

39. Compute the larger root of $x^2 = x + 1$.

40. Mike is preparing for the USA Math Olympiad. If he solves p problems on a certain day, then he will do $2 \cdot p + 1$ problems the next day. If Mike solves one problem the first day, how many problems will he do on the tenth day?

41. Let $A = (0, 0)$, $B = (4, 0)$, $C = (0, 3)$, and $D = (5, 5)$. What is the length of the shortest path which passes through the points A, B, C, and D?

42. The math team consumes 120 cans of Coca Cola and 199 cans of Canada Dry. Each math team member drinks at least one and at most three cans of Coca Cola as well as at least two and at most five cans of Canada Dry. If the math team has at least m and at most M members, what is $M - m$?

43. Two positive integers are relatively prime if they share no factors other than 1. How many positive integers less than 100 are relatively prime to 100?

44. John has 30π meters of wire. In meters, what is the largest possible area the wire can encompass if the two endpoints of the wire must touch?

45. Kevin has three identical white marbles and four identical black marbles. He puts them in a bag and randomly draws them out one by one without replacing them. On a sheet of paper, he records the sequence of colors (ex: BBWWWBB). How many such sequences are possible?

46. A rectangle with the area of 100 square feet has a circle inside it that lies tangent to 2 opposite sides and intersects the other two sides four times. If the area of this circle is 400π square feet, what is the length of the rectangle's shorter side?

47. An equilateral triangle is inscribed in a circle O with radius 6. Find the biggest possible radius of a circle that lies completely outside of the triangle and completely inside O.

48. Brian has a collection of stones. Each weighs a whole number of pounds. By combining stones from the set, he can make any whole number of pounds from 1 to 63 pounds, inclusive. What is the fewest number of stones that Brian can have?

49. Jordan thinks of a twelve-digit number such that the sum of any three consecutive digits is 17. If he divides number by 100, the remainder is 17. What is the leftmost digit?

50. Find the smallest positive integer n such that 120 divides $14n$.