Bergen County Academies Math Competition - 6th Grade

General Rules

- Calculators are not allowed.
- This is an individual test, so you may not communicate with anyone else taking it.
- Once time begins, we will not answer any questions about the problems.
- You will have 90 minutes to solve 50 problems. Once time is called, you must put down your pen or pencil and stop working.
- Scores will be posted on the website within a couple of days. Your score will appear next to your identification number.

Specifics

- You may use space on your test paper and additional scrap paper to do work. Your answers must be written on the answer sheet. We will not look at answers written on your test paper.
- Each problem has only one answer. If you put more than one answer for a problem, you will be marked wrong. When changing an answer, be sure to erase or cross out completely.
- Write legibly. If the graders cannot read your answer, it will be marked incorrect.
- Fractions should be written in lowest terms. For example, if the answer is $\frac{1}{2}$, then $\frac{2}{4}$ will not be accepted although the two fractions are numerically equal.
- All other answers should be written in simplest form.
- If a unit is indicated in the problem, the answer must be given in that unit. For instance, if the problem asks for the answer in hours, you cannot give your answer in minutes. Furthermore, you don't need to write the unit, as the graders will assume your answer is in the units asked for in the problem.
- There is no penalty for guessing.
- Ties will be broken based on the number of correct responses to the last ten questions. If a tie remains, then the correct responses to the last five questions will break the tie.
- We will announce how much time is remaining often during the test.

1. Compute $4 \times (0.00002 + 0.00020 + 0.00200 + 0.02000 + 0.20000 + 2.0000) + .00002$.

2. What is $1 + 1 \times (1 + 1 \times (1 + 1 \times (1 + 1 \times 1)))$?

3. Given $x\clubsuit y = \dfrac{2x + 3y}{4y - 2x}$. Compute $6\clubsuit(2\clubsuit 3)$.

4. 1, 1, 2, 3, 5, 8, ... What is the tenth number of this sequence?

5. How many prime numbers are less than 20?

6. How many positive integers less than 100 are not multiples of 2 or 3?

7. Find the sum of the counting numbers from 1 to 26 inclusive.

8. A triangle is called *isosceles* if two of its sides have the same length. Triangle ABC is isosceles with angle $A = 50°$. What is the sum of all distinct possible values of angle C?

9. Charlie the Unicorn is trying to climb up Candy Mountain. He climbs 3000 feet every day but falls 1200 feet every night when he sleeps. If the mountain is 10800 yards tall, how many days does it take Charlie to climb up the entire mountain?

10. Triangle ABC is similar to triangle DEF. If $\overline{AB} = 3$, $\overline{BC} = 4$, and $\overline{DE} = 6$, find the length of $\overline{EF}$.

11. How much money does Hannibal save after eating at Chili's if he applies tax on tip on meal instead of applying tip on tax on meal. Assume Hannibal spent 20 dollars on his meal, gave 20% tip, and paid 10% tax.

12. Bob has a prime number. Jane subtracts 35 from it, and the result is still a prime number. Find Bob's prime number.

13. You flip a coin 6 times. What is the probability that all 6 flips are tails?

14. A prime number is a positive whole number whose only positive divisors are 1 and itself. For instance, 2, 3, 5, and 7 are examples of prime numbers. What is the largest prime number smaller than 100?

15. Jongwhan, Mark, Robin and Kelly have jobs in Flatland. Jongwhan works every 5 days as a camp counselor. Mark works every 3 days as a cashier. Robin works every 6 days as a chef. Kelly works every 7 days as a computer technician. Today they are all working at their jobs. In how many days from today will they next be working on the same day again?

16. Evaluate: $1 + 2 - 3 + 4 + 5 + 6 - 7 + 8 + 9 + 10 - 11 + \ldots + 97 + 98 - 99 + 100$.

17. How many integers n are there such that $n = n^3$?

18. Steven takes a test where his raw score can be calculated by the following formula: $c - \frac{w}{4}$, where c is the number correct and w is the number wrong (answers left blank give 0 points). If Steven's raw score on a 100 question test was 57, what is the maximum number of problems Steven could have answered correctly?

19. Isabel is doing back to school shopping. She must buy three packs of paper for every two binders and one pack of pens for every pack of paper she buys. Each binder costs \$3.00, each pack of paper costs \$2.00, and each pack of pens cost \$1.00. If each of Isabel's 10 teachers requires her to buy a binder, how much money does she spend? (Answer in dollars)

20. What is the 2010th term of the sequence $1, 9, 6, 3, 1, 9, 6, 3, 1, 9, 6, 3, 1, 9, 6, 3, \ldots$?

21. A new ice cream store, Amazing Ice, has opened in town. An order of ice cream is made by choosing a flavor, topping and cone. At Amazing Ice, a customer can choose from 29 flavors, 15 toppings and 3 cones. The old ice cream store in town, Country Creamery, still thinks it has more orders, with 27 flavors, 19 toppings and 2 cones. How many more orders does the ice cream store with more orders have than the one with less?

22. Paul is trying to wrap his friends locker for her birthday. The locker is 5 feet tall and 9 inches wide. However, he needs to cut out a 4 in $\times$ 4 in square along the edge for the lock. If Paul can cut 3 inches of wrap per second, and his special wrapping paper is five feet long and five feet wide, what is the shortest time needed for Paul to cover the locker, assuming it takes 1 minute to tape the wrapping paper to the locker? Answer in seconds.

23. Find the sum of the reciprocals of the factors of 20.

24. If the sum of the costs of a bag of Corn Chips and a bag of Popcorn is 87 cents, a bag of Popcorn and a bag of Frosted Flakes is 76 cents, and a bag of Corn Chips and a bag of Frosted Flakes costs 99 cents, what is the total price, in cents, for a bag of each kind?

25. Three boys run around a track with constant speed. They start at the same point and then end at the same point 48 minutes later. The fastest one ran 6 minutes per lap, and the slowest one ran 8 minutes per lap. Given that the three boys each run at a different speed, how long did it take the middle one to run a lap?

26. What is the smaller angle formed between the minute hand and the hour hand at 6:09?

27. Jenny and Heejin decide to have a 5 mile running competition. Jenny runs at a speed of 110 yards every 30 seconds for 3 miles, then walks the rest of the way at a speed of 55 yards every 30 seconds the rest of the way. Heejin runs at a speed of 5 miles per hour. Who wins? (1760 yards=1 mile)

28. Alex, Jongwhan, and Michael each claims that he is the fattest among them. In order to find the truth, they decide to step on the scale two at a time. Alex and Jongwhan together weigh 210 pounds, Jongwhan and Michael together weigh 205 pounds, and Michael and Alex together weigh 221 pounds. Find how much Jongwhan weighs.

29. Mike is thinking of a number that leaves a remainder of 9 when divided by 16. What remainder does Mike's number leave when divided by 4?

30. If Chris has a colony of bacteria that doubles in population every half-hour, how many bacteria will he have at the end of four hours if he starts with 2 bacteria?

31. How many $3 \times 3 \times 3$ blocks does it take to completely fill a $12 \times 15 \times 18$ box?

32. Kelly is rowing a boat at a constant speed of 5 mph. She drops her barbie in the water, and the doll swims in the same direction at 60 mph. In 3 hours, how far will Kelly be from the barbie?

33. Ben Llama decides to participate in a 10-mile marathon. He begins the race running at 6 mph. Unfortunately, 30 minutes into the race he begins walking at 1 mph. He continues at this pace until the last half-mile, at which point he sprints to the finish line at 10 mph. How long did he take to complete the entire race? Answer in minutes.

34. At the Bergen County Academies, a chess club consists of 8 students. A team must be formed by choosing 4 students from this club. How many possible teams can be formed?

35. For some nonnegative reals x, y, z, we have $x + y + z = 80$ and $2x + 3y + 4z = 180$. If M and m are the maximum and minimum values for $100x + 80y + 50z$, find $M - m$.

36. Rectangle $ABCD$ has lengths $\overline{AB} = 15$ and $\overline{BC} = 10$. E is a point on $\overline{AD}$ such that $\overline{AE} = 1$. Find the area of triangle BCE.

37. In Dr. Nevard's Topics in Advanced Mathematics class, six people are taking a test. The possible scores are integers between 0 and 6, inclusive. If the median of the scores is 0.5, the range of the scores is 6, and the mean of the scores is 1.5, write down all the students' scores in increasing order.

38. How many positive integers k are there for which $64 - k$ is the square of an integer?

39. An $n \times n$ magic square is a grid containing the numbers 1 through n^2, such that the sums of the numbers in each row, column, and diagonal of the magic square are the same. Find the sum of the numbers in any given row of a 5 by 5 magic square.

40. If $x^2 - 4x + 3 = 0$, find $x + \frac{3}{x}$.

41. There are 69 kids in a room where $\frac{1}{3}$ of the kids are boys. Everyone is required to eat a cupcake. 40 kids said they prefer chocolate frosting. 8 girls want vanilla frosting. How many boys want chocolate frosting?

42. If x is real, find the smallest value that $x^2 - 2x - 2$ can be.

43. In the barn there are 70 animals: some are chickens and the rest are pigs. Jason counts 200 legs. Assume all pigs have four legs and all chickens have two. How many pigs are in the barn?

44. What is the smallest number of equilateral triangles of side length 0.99 that are required to completely cover up an equilateral triangle of side length 1?

45. A sequence of numbers generated accordingly to a pattern begins as follows: $1, 2, 4, 7, 11, 16, 22, \ldots$ What is the difference between the 2010th term and the 2011th term?

46. The radius of a circle is twice the edge of a cube. If the surface area of the cube is 54, find the area of the circle.

47. When 1 liter of a 40% vinegar solution is mixed with 2 liters of an 80% vinegar solution and 1 liter of water, what percent of the resulting solution is vinegar? Round your answer to the nearest tenth.

48. Robert can paint a fence in 1 hour by himself, and Mike can paint a fence in 2 hours by himself. How many minutes does it take Robert and Mike to paint a fence together?

49. Find the largest factor of 123,456 that is less than 123,456.

50. If x and y are positive integers, and $x + y + xy = 20$, what is the value of $x + y$?