Bergen County Academies Math Competition - 7th Grade

General Rules

- Calculators are not allowed.
- This is an individual test, so you may not communicate with anyone else taking it.
- Once time begins, we will not answer any questions about the problems.
- You will have 90 minutes to solve 50 problems. Once time is called, you must put down your pen or pencil and stop working.
- Scores will be posted on the website within a couple of days. Your score will appear next to your identification number.

Specifics

- You may use space on your test paper and additional scrap paper to do work. Your answers must be written on the answer sheet. We will not look at answers written on your test paper.
- Each problem has only one answer. If you put more than one answer for a problem, you will be marked wrong. When changing an answer, be sure to erase or cross out completely.
- Write legibly. If the graders cannot read your answer, it will be marked incorrect.
- Fractions should be written in lowest terms. For example, if the answer is $\frac{1}{2}$, then $\frac{2}{4}$ will not be accepted although the two fractions are numerically equal.
- All other answers should be written in simplest form.
- If a unit is indicated in the problem, the answer must be given in that unit. For instance, if the problem asks for the answer in hours, you cannot give your answer in minutes. Furthermore, you don't need to write the unit, as the graders will assume your answer is in the units asked for in the problem.
- There is no penalty for guessing.
- Ties will be broken based on the number of correct responses to the last ten questions. If a tie remains, then the correct responses to the last five questions will break the tie.
- We will announce how much time is remaining often during the test.

1. Divide 20 by $\frac{1}{2}$ and add 30. What is the resulting number?
2. Find $20-(20-(20-(20-(20-(20-(20))))))$.
3. If the width of any rectangle is increased by 10% and the length is reduced by 10%, by what percentage does the area of the rectangle decrease?
4. A and B are points on a circle with center O. If $\overline{AO} = x+3$, and $\overline{BO} = 3x$, what is x?
5. It takes 3 frogs 3 minutes to eat 3 flies. How many minutes does it take for 6 frogs to eat 6 flies?
6. How many factors does the number 2010 have?
7. A line with slope 4 intersects a line with slope 7 at the point $(10, 28)$. What is the distance between the x-intercepts of these two lines?
8. If $\sqrt{1000}$ is written in the form $a\sqrt{b}$, where a and b are integers, and b is not divisible by any perfect square, what is b?
9. Find the sum of all digits A that make the number $12A4$ divisible by 3.
10. Find the smallest positive integer that is divisible by both 24 and 21.
11. Bob writes down the numbers from 1 through 20. Find the number of digits Bob uses.
12. Jacobs sister has successfully snuck into his room and has gotten a hold of his stash of Twilight "gear." She plans on throwing them out one by one. She notices that he has 2 posters, 3 books, 1 keychain, and 3 plastic action figures. In how many ways can she throw them out if all 3 action figures must go first? She can only dispose of them one at a time and each item is distinct.
13. What is the last digit of 1337^{101}?
14. If $x+2y+2z=3$ and $2x+y+z=6$, find $x+y+z$.
15. A 30-60-90 right triangle has a hypotenuse of length 26. Find the length of the shorter leg.
16. Two shuttle buses are heading towards each other. They both travel at 40 mph. There is a butterfly fluttering at a constant speed of 45 mph between the shuttles. He starts from the center and flies in one direction until he hits the front of a shuttle. Immediately, he switches direction and continues to fly. This continues to occur until the trains meet. How far has the butterfly flown if the buses were originally 160 miles apart?
17. A rocket ship blasts off to the Moon at 8:52AM. It takes it 4 hours and 37 minutes to arrive there. Austin, the astronaut takes a walk on the Moon's surface for 2 hours and 42 minutes. It takes twice as long to get back to Earth than it takes to get to the Moon. What time is it when the rocket ship lands back on the Earth?
18. In a room of 10 people, everybody shakes hand with everyone else exactly once. How many handshakes were there in total?
19. If it takes Albert 6 hours to mow the lawn alone and Alex 4 hours to mow the lawn alone, find the amount of time it will take for them to mow the lawn together.
20. In how many ways can we put Amol, Evan, Wenyu, David, and Gabriel in a line if David cannot stand at either the front or the end of the line?
21. What is the sum of the interior angles of an octagon, a figure with 8 sides?
22. In how many distinguishable ways can the letters in the word "puppy" be arranged?
23. For how many integer values of x is $x^2-9x+20$ prime?
24. Alex's drawer has 36 white socks, 9 purple socks, 15 pink socks, and 17 orange socks. Alex blindly reaches into his drawer and draws out some number of socks. What is the smallest number of socks Alex will need to draw to guarantee that he has 2 socks of the same color?

25. Two perpendicular diameters of a circle are drawn, splitting it into four equal sections. If one of the sections has area 16pi, what is the circle's diameter?

26. If $f(x) = 9x^2 + 5$ and $g(x) = \dfrac{5x + 12}{x^3}$, find $f(g(3))$.

27. Find the measure of the smaller angle formed by the hour and the minute hands of an analog clock at 1:37 PM.

28. The time is now 1:37 PM. In an analog clock, the minute hand will catch up to the hour hand in exactly x minutes, where x is positive and as small as possible. Find x.

29. If A, B, and C are three distinct non-collinear points (not on the same line), how many parallelograms can be formed using A, B, C, and a fourth point?

30. There is a right triangle ABC where $\angle B = 90°$, $\overline{AB} = 6$ and $\overline{BC} = 8$. A line segment is drawn from B so that it is perpendicular to side AC at point D. What is the length of this line segment?

31. In a single elimination chess tournament (a person is eliminated once he loses), how many games will it take to decide a winner if there are 2010 participants?

32. $n!$ is defined to be the product of all of the positive integers less than or equal to n. For example, $3! = 3 \times 2 \times 1$. Find the last digit of $1! + 2! + 3! + \cdots + 10!$.

33. If the sum of the mean, median, and mode of the numbers $4, 5, 6, 9, 9, 9, x$ is 23, find x.

34. Weili is tethered to a wall at Point A by a rope which is five feet long. Point A is located along Wall B three feet from the intersection of Wall B and Wall C, which are perpendicular rays originating from their intersection. If the Weili can roam anywhere his rope allows him to, what is the total area he can traverse?

35. Let x, y, z be nonzero rational numbers where $x : y : z = 2 : 3 : 5$ and $x^2 + y^2 + z^2 = xyz$. Find the value of $x + y + z$.

36. The volume of a cube is equal to its surface area. What is its side length?

37. Alex has to walk down a hill to get to school every day, and up that hill to return from school to home. On his way to school, he travels at 6 miles per hour. On his way back from school, he travels 4 miles per hour. Find his average speed on his round trip, in miles/hour.

38. Two passenger trains traveling in opposite directions meet and pass each other. Each train is 50 meters long and is traveling at 36 kilometers per hour. How many seconds after the front parts of the trains meet will their rear parts pass each other?

39. Dan ran 3 miles in 19 minutes, walked 4 miles in 40 minutes, and then ran another 3 miles in 21 minutes. What was its average speed for the whole trip in miles per hour?

40. There are three fields with the same area and the same rate of grass growth. There are also cows in these fields, which eat grass at a constant rate. In the first field, there are 9 cows, and they ate all the grass in the field in 14 days. In the second field, there are 11 cows, and they ate all the grass in the field in 10 days. In the third field, there are 14 cows. How many days will it take for them to eat all the grass in the field?

41. The ratio of the area of a circle to the circumference of a circle is 5 : 1, what is the radius of this circle?

42. Given that $1 = 1^2$, $1 + 3 = 2^2$, and $1 + 3 + 5 = 3^2$, find $\sqrt{1 + 3 + 5 + \cdots + 4001}$.

43. Find the units digit of $13^{2010} \times 11^{2011} \times 7^{2012}$.

44. Suppose a clock is completely accurate, but only has an hour hand. What time is it when it is pointing at 22 minutes? (Disregard AM/PM)

45. What is the least positive integer that leaves a remainder of 1 when divided by 2, leaves a remainder of 2 when divided by 3, leaves a remainder of 3 when divided by 4, and leaves a remainder of 4 when divided by 5?

46. A palindrome is a number that is read the same way forward or backward. For example, the numbers 14541 and 1221 are palindromes, but the number 130531 is not. How many three-digit numbers are palindromes? (A three-digit number cannot start with the digit 0.)

47. Convert 20110 – a number written in base 3 – to base 10.

48. A cow starts on vertex A of square $ABCD$. Every second it moves along a side of the square to an adjacent vertex. However, each of its moves is completely random. After 11 seconds, how many ways can the cow end up on vertex B?

49. Let $f(x) = \dfrac{1}{(2x-1)(2x+1)}$. Find $f(1) + f(2) + \cdots + f(2010)$.

50. Compute $i^{F_1} + i^{F_2} + \cdots + i^{F_{2009}} + i^{F_{2010}}$ where $i = \sqrt{-1}$ and $\{F_n\}$ is the Fibonacci sequence with $F_n = F_{n-1} + F_{n-2}$ and $F_1 = F_2 = 1$.