1. $164 \div 4 = \boxed{41}$.

2. Danny Kim is 5ft 3in tall, so he is $5 * 12 + 3 = 63$ inches tall. He wants to grow until he is 2 inches taller than Jeremy Lin, who is 75 inches tall, so he wants to grow until he is $75+2 = 77$ inches tall. Therefore, he needs to grow $77 - 63 = \boxed{14}$ more inches.

3. $\frac{1}{4} \cdot (\frac{1}{4} + \frac{1}{4}) = \boxed{\frac{1}{8}}$.

4. If Dr. Abramson buys 1 slice of pizza for all 160 students, then he will spend $\$1.50 \cdot 160 = \$240$. If he buys whole pies, he will require $\frac{160}{8} = 20$ pizzas, and will spend $20 \cdot \$9 = \$180$. Therefore, he will save $240 - 180 = \boxed{\$60}$.

5. Kevin and Chester place higher than Dominic, so neither Kevin nor Chester can be 4th place. Ryan does not finish in the bottom 2, so he cannot be 4th place either. Therefore, $\boxed{\textit{Dominic}}$ must be in 4th place.

6. Since 1126 is even, one of the primes is 2. Since we are told that the number is the product of exactly two primes, we know that the other prime is $\frac{1126}{2} = 563$ without any computation. Therefore, the sum is $563 + 2 = \boxed{565}$.

7. Cross multiply to get $84x = 28 \cdot 36$. Notice that $84 = 3 \cdot 28$, so $28 \cdot 3x = 28 \cdot 36$, so $3x = 36$. Therefore, $\boxed{x = 12}$.

8. The 200th positive even number is just $2 \cdot 200 = 400$. Since the nth positive odd number is just one less than the nth positive even number (ex. $1 = 2 - 1$, $3 = 4 - 1$, etc.), the 200th positive odd number is $400 - 1 = 399$. Their sum is $399 + 400 = \boxed{799}$.

9. Add the two equations to get $4x = 56$, so $x = 14$.

10. $111111^2 = 12345654321$

11. Solution 1: Let x be the middle integer. We have that $(x-8) + (x-7) + \cdots + (x+7) + (x+8) = 0$. This simplifies to $x + (x+1) + (x-1) + \cdots + (x+8) + (x-8) = 17x = 0$, meaning that $x = 0$. So the smallest of the consecutive integers is $x - 8 = \boxed{-8}$.

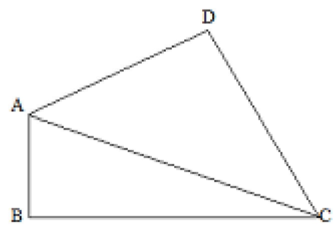
    Solution 2: Since 17 consecutive integers sum to 0, some of them must be negative and some must be positive, so 0 must be one of the numbers included. In fact, since they are consecutive integers that sum to 0, there must be an equal number of positive and negative numbers, or $\frac{(17-1)}{2} = 8$ numbers each. Therefore, the smallest of these 17 consecutive integers is the 8th negative integer, or $\boxed{-8}$.

12. If the number $8X54$ is divisible by 3, then by the divisibility rule, its digits must add up to a multiple of 3. The sum of its digits is $8 + X + 5 + 4 = 17 + X$. Therefore, X can be 1, 4, or 7, of which the largest possible answer is $\boxed{7}$

13. Any number that is of the form $0.abcabcabc\cdots$ will be of the form $\frac{abc}{999}$. Therefore, we can set up the equation: $\frac{4}{37} = \frac{abc}{999}$. Cross multiplying and solving, we obtain that $abc = 108$. Therefore, $\boxed{a + b + c = 9}$.

14. The smallest number of items Fritz can buy to be able to be divided into fourths, fifths, and sixths, is simply the LCM (least common multiple) of 4, 5, and 6, which is $4 \cdot 5 \cdot 3 = \boxed{60}$.

15. Arthur scored $3 \cdot 30 = 90$ points with three-point shots and $0.80 \cdot 100 = 80$ points with free throws. Therefore, he scored a total of 170 points. The total possible points he could have scored is $3 \cdot 45 + 100 = 235$. Therefore, the answer is $\frac{170}{235} = \boxed{\frac{34}{47}}$.



16. In the diagram above, we can see that by drawing segment $\overline{AC}$, we get a hypotenuse of both right triangles ABC and CDA. To confirm this, we note that $AB^2+BC^2 = 7^2+24^2 = CD^2+AD^2 = 15^2 + 20^2 = 25^2$. Therefore, $AC = 25$ and we can find the area of the quadrilateral by adding the areas of the two triangles, which is $\frac{1}{2}(7)(24) + \frac{1}{2}(20)(15) = 84 + 150 = \boxed{234}$.

17. This 10 day period will include 2 Sundays, 1 Saturday, and 7 weekdays. Therefore, the company will sell $3 \cdot 10 + 7 \cdot 5 = \boxed{65}$ shirts in all.

18. Work backwards. Notice that when $b = 1$, $a\#b$ simplifies to $a - a = 0$. Since we are being asked to find (something)#1, this is just $\boxed{0}$.

19. We know, from 1 across and 1 down, that a two-digit perfect cube and a two-digit perfect square must have the same tens digit. The only options for the tens digit are therefore 2 (27 and 25) or 6 (64 and 64). However, if the tens digit is 6, then by 3 across we must have a multiple of 13 that begins with a 4. However, no such multiple exists, so the tens digit cannot be 6. Instead, it must be 2. We are now looking for a two-digit multiple of 13 that begins with a 5, and a two-digit multiple of 8 that begins with 7. We see that putting 2 in the last two boxes, giving 72 and 52, satisfies the criteria. Our final crossword looks like this:

2	7
5	2

20. See #19

21. See #19

22. See #19

23. Add 1 to both sides. Notice that the left-hand side factors as $(x + 1)^3$, and the right-hand side is $125 = 5^3$. Therefore, $x + 1 = 5$, so $\boxed{x = 4}$.

24. Each sequence is composed of a 1 followed by one more 2 than the previous sequence had before it. Therefore, we know that the first number is a 1, and the second 1 is two numbers after (3rd), the next one is three numbers away (6th), and so on. Continuing this, we see that there are 10 ones in this pattern (occurring in positions 1, 3, 6, 10, 15, 21, 28, 36, 45, 55). The remaining 53 numbers are 2's, so the sum of the first 63 terms is $53 \cdot 2 + 10 = \boxed{116}$.

25. Say there were x Doduos and y Dodrios. There are 291 heads in total, so $2x + 3y = 291$. There are 222 feet in total, so $2x + 2y = 222$. Subtracting the second equation from the first, we get $y = 291 - 222 = 69$. We can plug in $y = 69$ into either of the two equations to get $\boxed{x = 42}$.

26. There are two options: either Andrew chooses two boys, or he chooses one boy and one girl. For the former option, he has $\frac{5 \cdot 4}{2} = 10$ ways to choose two members for his band, while for the latter option, he has $5 \cdot 4 = 20$ ways to choose two members. Therefore, he has $20 + 10 = \boxed{30}$ different pairs of people to choose from.

27. Using the formula for an infinite geometric series, $\frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3} + \cdots = \frac{\frac{1}{a}}{1 - \frac{1}{a}}$. Multiplying the top and bottom by a, we have that $\frac{1}{a-1} = \frac{4}{5}$. We can solve for a to get $a = \frac{9}{4}$. Let $S = \frac{1}{a^2} + \frac{1}{a^4} + \frac{1}{a^6} + \cdots$. Then $aS = \frac{1}{a} + \frac{1}{a^3} + \frac{1}{a^5} + \cdots$. Therefore, $aS + S = \frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3} + \cdots = \frac{4}{5}$. Plugging in $a = \frac{9}{4}$, we have that $\frac{9}{4} \cdot S + S = \frac{13}{4} \cdot S = \frac{4}{5}$. Solving for S, we get $S = \boxed{\frac{16}{65}}$.

28. Since there is absolutely no way to feasibly calculate $5^{2012!}$, we must look for a pattern. The last three digits of 5^1 are 005, for 5^2 they are 025, for 5^3 they are 125, and for 5^4 they are 625. Then we see that $5^5 = 3125$, which ends in 125, and that means that 5^6 will end in 625. This is a pattern that repeats with period 2, so all large odd powers of 5 will end in 125, while all large even powers of 5 will end in 625. Since 2012! is even, the last three digits of $5^{2012!}$ are $\boxed{625}$.

29. For the sake of argument, let us suppose that the distance between his house and his school is 240 miles. If we let t be the time in hours that Dennis took to get to school, then he drove $\frac{t}{2}$ for 40 mph and $\frac{t}{2}$ for 60 mph. Since distance is product of time and velocity, we get that $240 = \frac{t}{2} \cdot 40 + \frac{t}{2} \cdot 60 = 50t \implies t = 4.8$. Thus, it took him 4.8 hours to get to school. Secondly, on the way home, Dennis drove 120 miles with 40 mph, and another 120 miles with 60 mph. Hence, it took him $\frac{120}{40} + \frac{120}{60} = 5$ hours to get back home. Finally, since he took $5 + 4.8 = 9.8$ hours to drive a total of $2 \cdot 240 = 480$ miles, his average speed for the entire trip is $\frac{480}{9.8} = \boxed{\frac{2400}{49}}$ mph.

30. Shift the 1 to the right side, giving $x^{72} + x^{24} + x^4 + x^2 = -1$. Now each term on the left-hand side is a perfect square, and perfect squares are always nonnegative. Therefore, the sum of four perfect squares cannot be negative, and therefore there are $\boxed{0}$ solutions to this equation.

31. Let the smaller number be $ABCD$, then the digits of the larger number are $A+1, B+1, C+1, D+1$. Therefore, the larger number is 1111 greater than the smaller number. Let the smaller number be x. Then $x + (x + 1111) = 5823$. Solving for x, we have $x = 2356$. Therefore, the larger number is $2356 + 1111 = \boxed{3467}$.

32. First, square $a+b=5$ to get $(a+b)^2 = a^2+2ab+b^2 = 25$. Then, we can subtract $2ab=14$ from this equation to get $a^2+b^2=11$. Now we square a^2+b^2 to get $\left(a^2+b^2\right)^2 = a^4+2a^2b^2+b^4 = 121$. However, since $2a^2b^2$ is just $2(ab)^2=98$, we can subtract that from the equation to get $a^4+b^4 = 121-98 = \boxed{23}$.

33. In a best scenario, any two lines would intersect each other exactly once. Therefore, there are $\binom{10}{2} = 45$ intersections among the 10 lines. Each line could intersect each of the 2 circles twice, so there are at maximum 20 intersections of lines and circles. Two circles can intersect each other at max 2 times. Therefore, the maximum number of intersections in total is $45+20+2 = \boxed{67}$.

34. For a positive integer of the form $p_1^{e_1}\dots p_n^{e_n}$, the number of factors it has is $(e_1+1)\dots(e_n+1)$. Therefore, for the number to have exactly 12 factors, the expression $(e_1+1)\dots(e_n+1)$ must equal 12. Prime factoring 12, we see that we can satisfy this condition in many ways:

    Case 1: The number has only one factor, i.e. it is of the form p^{e_1}. Then, in order for it to have 12 factors, we must have $e_1+1=12 \rightarrow e_1=11$. Therefore, it must be of the form p^{11}. The smallest prime p is 2, so the smallest number for this case is $2^{11}=2048$.

    Case 2: The number has two factors, i.e. it is of the form $p_1^{e_1}p_2^{e_2}$. Then, for this to have 12 factors, we must have $(e_1+1)(e_2+1)=12$. This can be done in two ways: either $p_1^3p_2^2$ or $p_1^5p_2^1$. Either way, we will choose $p_1=2$ and $p_2=3$ because those are the two smallest primes, and we want the smallest prime possible for the largest exponent. These two give $8\cdot 9=72$ and $32\cdot 3=96$, respectively.

    Case 3: The number has three factors, i.e. it is of the form $p_1^{e_1}p_2^{e_2}p_3^{e_3}$. Then, for this to have 12 factors, we must have $(e_1+1)(e_2+1)(e_3+1)=12$. This can only be done in one way: if $e_1=2$, $e_2=1$, and $e_3=1$ (letting any $e_n=0$ reverts to one of the earlier cases). Therefore, the number must be of the form $p_1^2p_2^1p_3^1$. Following the rule of giving the largest exponent the smallest possible prime, we get the smallest possible value of that expression by setting $p_1=2$, $p_2=3$, and $p_3=5$, giving us a final answer of $4\cdot 3\cdot 5=60$.

    Of the three cases, the smallest possible number with exactly 12 factors is $\boxed{60}$.

35. Let this 3-digit number be abc. Then the equation we can write is $a^2+b^2+c^2=2(a+b+c)$. By subtracting $2a+2b+2c$ from both sides, and adding 3 to both sides, we can group terms to get: $(a-1)^2+(b-1)^2+(c-1)^2=3$. The only way for 3 squares to sum to 3 is $1+1+1$. Therefore, $(a-1)^2=1$. Since none of the digits are 0, we have that $a=2$. Similarly, we obtain that $a=b=c=2$. Therefore, our number is $\boxed{222}$.

36. We split this problem up into two cases: the first, when the animals on both ends are both dogs, and the second when they are both cats.

    Case 1: Both animals are dogs. Then we have to arrange the remaining 5 identical cats and 2 identical dogs in a row between the two dogs. There are $\frac{7!}{5!2!}=21$ ways to do this.

    Case 2: Both animals are cats. Then we have to arrange the remaining 3 identical cats and 4 identical dogs in a row between the two cats. There are $\frac{7!}{3!4!}=35$ ways to do this.

    Adding the two cases together, there are a total of $35+21=\boxed{56}$ arrangements of cats and dogs that satisfy the criteria.

What Kelvin Rolls	What AJ Can Roll
1	1, $\boxed{2, 3, 4, 5, 6, 7, 8, 9, 10}$
2	1, 2, 3, $\boxed{4, 5, 6, 7, 8, 9, 10}$
3	1, 2, 3, 4, 5, $\boxed{6, 7, 8, 9, 10}$
4	1, 2, 3, 4, 5, 6, 7, $\boxed{8, 9, 10}$
5	1, 2, 3, 4, 5, 6, 7, 8, 9, $\boxed{10}$
6	1, 2, 3, 4, 5, 6, 7, 8, 9, 10

37. The way the table above is constructed lends itself to the solution. If Kelvin olls k, then for AJ's roll to be at least twice Kelvin's, AJ must roll a number between $2k$ and 10 inclusive. There are $10 - 2k + 1 = 11 - 2k$ such numbers. We note that if Kelvin rolls a 6, AJ's roll cannot be at least twice of that. Therefore, we are summing $11 - 2k$ for $1 \leq k \leq 5$. If we write this out, we see that we are just adding $9 + 7 + 5 + 3 + 1 = 25$. Hence, the answer is $\frac{25}{60} = \boxed{\frac{5}{12}}$

38. We first prime factor 3600 as $60^2 = 2^4 \cdot 3^2 \cdot 5^2$. For a factor of this to be a perfect square, all of its prime factors must be raise to an even power. This means that the power of 2 can be either 0, 2, or 4 (yes 1 is a perfect square factor too!). The power of 3 can be either 0 or 2, and the power of 5 can be either 0 or 2. Therefore, there are $3 \cdot 2 \cdot 2 = \boxed{12}$ perfect square factors of 3600.

39. If a number is congruent to 1 mod 3 and 1 mod 5, then it must be congruent to 1 mod 15. We can list the first few numbers that leave a remainder of 1 when divided by 15: 1, 16, 31, 46, 61, 76, 91, 106, ?. We see that $\boxed{91}$ is the first number that is divisible by 7.

40. Call these two numbers x and y, with $x > y$. We have that $x - y = 9$, or $x = y + 9$, and that xy is as small as possible. Substituting $x = y + 9$ into the second equation gives us that $y(y + 9)$, or $y^2 + 9y$, is as small as possible. Since this is a quadratic, its minimum occurs when $y = -\frac{b}{2a} = -\frac{9}{2\,(1)} = -4.5$. However, since x and y are integers, we can simply choose y to be the integer closest to -4.5, which is either -4 or -5 (we will see that they both give the same product). For the two choices of y listed above, plugging them into the first equation gives $x = 5$ and $x = 4$, respectively. Both of these pairs of x and y give a product of $-5\,(4) = -4\,(5) = \boxed{-20}$.

41. The triangular faces contribute $3 \cdot 8 = 24$ edges. The octagonal faces contribute $8 \cdot 6 = 48$ edges. However, each edge is counted twice since every edge is the meeting of two faces. Therefore, there are $\frac{24 + 48}{2} = \boxed{36}$ edges in total.

42. Solution 1: James can flip: 7 heads and 0 tails, 6 heads and 1 tail, 5 heads and 2 tails, or 4 heads and 3 tails. 7 heads: There is only one way for him to do this, and it happens with probability $\left(\frac{1}{2}\right)^7 = \frac{1}{128}$.

6 heads: There are $\binom{7}{6} = 7$ ways for him to do this, and it happens with probability $7\left(\frac{1}{2}\right)^7 = \frac{7}{128}$.

5 heads: There are $\binom{7}{5} = 21$ ways for him to do this, and it happens with probability $21\left(\frac{1}{2}\right)^7 = \frac{21}{128}$. 4 heads: There are $\binom{7}{4} = 35$ ways for him to do this, and it happens with probability $35\left(\frac{1}{2}\right)^7 = \frac{35}{128}$.

Therefore, the total probability James has of flipping more heads than tails is $\frac{1}{128} + \frac{7}{128} + \frac{21}{128} + \frac{35}{128} = \frac{64}{128} = \boxed{\frac{1}{2}}$.

Solution 2: Let the probability James flips more heads than tails be P_H, the probability that he flips more tails than heads be P_T, and the probability that he flips an equal number of heads as tails be P_E. Since these are the only three possibilities when flipping coins, $P_H + P_T + P_E = 1$. Furthermore, since James flips a fair coin, there is no difference between him flipping 4 heads and 3 tails or him flipping 4 tails and 3 heads. This is because the probability of flipping h heads is the same as the probability of flipping h tails. Thus, $P_H = P_T \to 2P_H + P_E = 1 \to P_H = \frac{1-P_E}{2}$. However, since James flips 7 coins, there is no way for him to flip an equal number of heads as tails, i.e. $P_E = 0 \to P_H = \boxed{\frac{1}{2}}$.

43. The first 2012?1337 = 675 digits will be 9, while the rest of the digits will be 0. Therefore, the digit-sum is $675 \cdot 9 = \boxed{6075}$.

44. In order to have exactly one patient with each condition, the doctor must also have one patient who is perfectly fine. Since the four patients can be afflicted with each status condition in any order, we must multiply the final probability by the number of orderings, which is just $4 \cdot 3 \cdot 2 \cdot 1 = 24$. The probability of having exactly one patient with each condition is therefore $24 \cdot \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{6} \cdot \frac{1}{4} = \boxed{\frac{1}{12}}$.

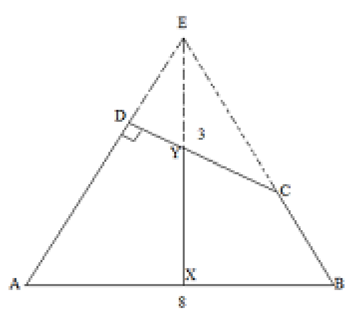
45. First, we find the radius of the incircle. It is known that $A = rs$ where A is the area of the triangle, r is the inradius, and s is the semi-perimeter. Using Heron's formula, we obtain that $A = \sqrt{27(27-17)(27-18)(27-19)} = 36\sqrt{15}$. The semi-perimeter is $s = \frac{17+18+19}{2} = 27$. Therefore, the inradius is $r = \frac{36\sqrt{15}}{27} = \frac{4\sqrt{15}}{3}$. Let the incircle be tangent to side PQ at point T, side QR at point V, and side RP at point U. Then $TI = \frac{4\sqrt{15}}{3}$. It is well known that $PT = \frac{PQ+PR-QR}{2} = \frac{17+19-18}{2} = 9$. (This can be seen by noting that

tangents from the same point to a circle are of equal length, i.e. $PT = PU$, $QT = QV$, and $RV = RU$.) Therefore, $(PI)^2 = (PT)^2 + (TI)^2 = 81 + \frac{80}{3} = \boxed{\frac{323}{3}}$.

46. We will solve this problem by complementary probability—finding the probability that the bug does **not** reach all three vertices in three seconds. There are only two ways the bug can achieve this: either the bug goes (to the following vertices in order) BAB or CAC. These two ways both have the same probability, $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$. Therefore, the probability that the bug does **not** reach all three vertices is $\frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$, and the probability that he does reach all three vertices is $1 - \frac{1}{4} = \boxed{\frac{3}{4}}$.

47. The formula for the volume of a tetrahedron with side length s is: $\frac{s^3\sqrt{2}}{12}$. Plugging in $s = \sqrt{2}$, we get that the volume is $\frac{4}{12} = \frac{1}{3}$. Let the side length of the cube be x. Then $x^3 = 1/3$ and $\boxed{x = \frac{1}{\sqrt[3]{3}}}$.

Proof that the volume of a tetrahedron with side length s is: $\frac{s^3\sqrt{2}}{12}$: Let the vertices of the tetrahedron be A, B, C, D. Let the foot altitude from D to face ABC be H. $(HO)^2 = (AD)^2 - (AO)^2 = s - (\frac{s}{\sqrt{3}})^2 = \frac{2}{3}$. Therefore, $HO = \frac{s\sqrt{2}}{\sqrt{3}}$. The area of ABC is $\frac{s^2\sqrt{3}}{4}$, so the volume of the tetrahedron is: $\frac{1}{3} \cdot \frac{s\sqrt{2}}{\sqrt{3}} \cdot \frac{s^2\sqrt{3}}{4} = \frac{s^3\sqrt{2}}{12}$.



48. Since the sum of the angles in a quadrilateral is 360°, we can use this fact to find that $\angle D = 360 - 150 - 60 - 60 = 90^\circ$, or a right angle. Now we extend $\overline{AD}$ and $\overline{BC}$ to meet at point E, forming an equilateral triangle ($\angle E = 180 - 60 - 60 = 60^\circ$). Since the angle bisector and perpendicular bisector in an equilateral triangle are the same, E, X, and Y are collinear. We notice that since $\angle EAX = 60^\circ$ and $\angle EXA = 90^\circ$, $\triangle AEX$ is a 30-60-90 right triangle, and $EX = AX\sqrt{3} = 4\sqrt{3}$. Now we only need to find the value of $\overline{EY}$, which is the hypotenuse of $\triangle DEY$. Since $\angle DEY = 30^\circ$ and $\angle DEC = 60^\circ$, $\triangle DEY$ and $\triangle DEC$ are also 30-60-90 right triangles. From $\triangle DEC$, we know that $CD = 3$, so $DE = \frac{CD}{\sqrt{3}} = \sqrt{3}$. We then use this new piece of information for $\triangle DEY$, noting that $DY = \frac{DE}{\sqrt{3}}$ and $EY = 2DY$, meaning that $EY = \frac{2DE}{\sqrt{3}} = 2$. Therefore, the length of $\overline{XY}$ is $EX - EY = \boxed{4\sqrt{3} - 2}$.

49. We will proceed by complementary counting. We notice that there are $\frac{7!}{3!} = 840$ ways to order the letters in BREEZES without restrictions. Now we notice that there are two ways to have a vowel with no consonants: <u>Case 1</u>: "EE" appears at the end. Consider this string "EE" as one letter "E". There are two ends we can place it at, and then $5! = 120$ ways to place the remaining 5 letters before or after it. Therefore, there are $2 \cdot 120 = 240$ arrangements that don't work in this case. <u>Case 2</u>: "EEE" appears somewhere in the arrangement. Consider this string "EEE" as one letter "E". There are then $5! = 120$ arrangements that don't work in this case. However, we have overcounted the times when "EEE" is at either end of the string, so we must add back that many. Considering "EEE" as a single string, it can go either in the front or at the back, and there are $4! = 24$ ways to arrange the rest of the letters. Therefore, we add back $2 \cdot 24 = 48$ arrangements. Our final answer is therefore $840 - 240 - 120 + 48 = \boxed{528}$.

50. First, we will find the number of 10-digit numbers that satisfy these criteria. The first digit can be any number from $\{1,\ 2,\ \ldots,\ 9\}$, whereas the remaining digits can be any remaining digit from $\{0,\ 1,\ 2, \ldots,\ 9\}$. Therefore, there are 9 choices for the first digit, $10 - 1 = 9$ choices for the second, 8 for the third, and so on. This gives us $9 \cdot 9!$ different 10 digit numbers that satisfy the criteria. Of these $9 \cdot 9!$ numbers, $9!$ of them begin with 1, $9!$ of them begin with 2, and so on, meaning that the average of the first digit is $\frac{1 \cdot 9! + 2 \cdot 9! + \cdots + 9 \cdot 9!}{9 \cdot 9!} = \frac{1 + 2 + \cdots + 9}{9} = 5$ (i.e. the first digit of the average 5). For each of the 2nd through 10th digits, the average is different because we include a 0, but the process is similar. Of the $9 \cdot 9!$ numbers, since 0 is never a first digit, $9!$ of them have 0 as their second digit. Of the remaining $8 \cdot 9!$ numbers, the process is the same as it was for the first digit. $\frac{8 \cdot 9!}{9} = 8 \cdot 8!$ of them have 1 as the second digit, $8 \cdot 8!$ of them have 2 as the second digit, and so on. Therefore, the average of each of the second through tenth digits is $\frac{9! \cdot 0 + 8 \cdot 8! \cdot 1 + 8 \cdot 8! \cdot 2 + \cdots + 8 \cdot 8! \cdot 9}{9 \cdot 9!} =$ $\frac{8 \cdot 8! \cdot (1 + 2 + \cdots + 9)}{9 \cdot 9!} = \frac{8 \cdot 45}{9 \cdot 9} = \frac{40}{9}$

But $\frac{40}{9}$ isn't a digit, so we can't just place it in each of the second through tenth digits like we did for 5 in the first digit. However, we can use expanded notation to our advantage. Placing $\frac{40}{9}$ in each of the second through tenth digits is the same as writing $\frac{40}{9} \cdot 10^8 + \frac{40}{9} \cdot 10^7 + \frac{40}{9} \cdot 10^6 + \cdots + \frac{40}{9} \cdot 10^0 = \frac{40}{9} \cdot (111111111) = 493827160$. Placing a 5 in front of this number from calculating the first digit gives us the final answer of $\boxed{5493827160}$.