2015 Joe Holbrook Memorial Math Competition
4th Grade Exam Solutions

The Bergen County Academies Math Team

October 11th, 2015

1. $13 + 12 + 10 + 7 + 4 + 3 + 1 = \boxed{50}$.
2. The number of water bottles he drinks in a day is the sum of the numbers of water bottles he drinks at each meal. $2 + 3 + 5 = \boxed{10}$.
3. When 2 people are standing on top of each other, their combined height is the sum of their individual heights. Therefore, the combined height in this case is $86 + 100 = \boxed{186}$
4. Because addition and subtraction are on the same level in the order of operations, we can move left to right: $2015 + 201 + 20 + 2 - 15 - 5 - \boxed{2218}$.
5. $5 \times 6 = \boxed{30}$.
6. (Thomas' weight) + (book's weight) = 350, so (Thomas' weight) = 350 − (book's weight) = $350 - 5 = \boxed{345}$.
7. First, we use order of operations: the term with multiplication/division, $4 \times \frac{15}{3}$ becomes $4 \times 5 = 20$. Adding yields $\boxed{25}$.
8. The Sun sets every evening, which comes after noon. Thus between noon Saturday and noon Thursday, Saturday, Sunday, Monday, Tuesday, and Wednesday all have evenings; this makes $\boxed{5}$ days.
9. Sixteen years ago, Sunshine was four years old, while Moonbeam was not yet born, so $\boxed{\text{Moonbeam}}$ was born second.
10. There are 11 fish, each of which eats 19 flakes, so the total amount that Nemo needs can be computed by $11 \times 19 = \boxed{209}$.
11. The first term of the sequence is 1, and each subsequent term is 3 more than the previous one, so the second term is $1+3 = 4$, the third term is $1+3+3 = 7$, and so on. Extending this, the nth term is $1 + 3(n-1)$, so the 100th term is $1 + 3(100-1) = 1 + 3(99) = 1 + 297 = \boxed{298}$.
12. If Sung Hyup was 15 years old 4 years ago, he is now 19 years old. Sunny is 3 years younger than him, so he is currently 16. In two years he will be $16 + 2 = \boxed{18}$.
13. If she bikes, then she takes $\frac{3 \text{ miles}}{12 \text{ mph}} = \frac{1}{4}$ hours = 15 minutes. If she walks, then she takes $\frac{3 \text{ miles}}{4 \text{ mph}} = \frac{3}{4}$ hours = 45 minutes. Riding a bike will be $\boxed{30}$ minutes faster.
14. There are going to be 6 bonsai-to-bonsai gaps (as gap count = $\frac{\text{total length}}{\text{gap length}}$). Each gap has 2 endpoints, so there are going to be $\boxed{7}$ trees total.

15. There are three ways to choose the cheese. Once Thomas has chosen the cheese, he now has four options for his cracker. Thus for each of the three cheeses, there are four cracker options, so the number of combinations is $3 \times 4 = \boxed{12}$.

16. If there are 2015 consecutive integers, then they must be $n-1007, n-1006, \ldots n-1, n, n+1, \ldots n+1006, n+1007$, for some n. When computing the sum, each $n-k$ is canceled out by $n+k$, so the total sum is $2015n$ and the mean is n, which is also the median of the list. The mean is 2, so $\boxed{2}$ is the median as well.

17. The cafeteria and the gym would be the furthest apart if they were on opposite sides of the dorm; the distance would then be $20 + 10 = 30$*hops*. On the other hand, the shortest distance would be if they were along a straight line on the same side of the dorm, and that distance would equal $20 - 10 = 10$. Thus the final answer is $30 - 10 = \boxed{20}$.

18. He did an hour and a half of math on Sunday; there are 60 minutes in an hour, so this is $60 \times 1.5 = 90$ minutes. He must do 30 minutes for each hour of video games, so the number of hours played can be computed by $\frac{90}{30} = \boxed{3}$.

19. The least positive multiple of 7 is $7 \times 1 = 7$, while the greatest less than 100 is $7 \times 14 = 98$. Thus there are $\boxed{14}$ multiples.

20. $5@6 = 5 \times 6 - 5 + 6 = 31$. $31@5 = 31 \times 5 - 31 + 5 = \boxed{129}$.

21. Every other integer, i.e. every one out two, is even, and every other one is odd. Thus from 1 to 200 there are $\frac{200}{2} = 100$ even numbers, and from 1 to 300 there are $\frac{300}{2} = 150$ odd numbers. Their difference is $\boxed{50}$.

22. A pyramid is the shape formed by a polygonal base and the line segments from each of the vertices of the base to another point in a different plane, called the *vertex* of the pyramid. If the base is octagonal, then there are 8 edges on the octagon and 8 edges from each of the octagon's vertices, for a total of 16. For the faces, there is the octoganal one, along with 8 triangular faces formed by the vertex-edges and the edges of the octagon, for a total of 9. Thus the answer is $16 + 9 = \boxed{25}$.

23. There are 2 hours and 29 minutes from 5:00 to 7:29. Each hour is 60 minutes long, so this is $2(60) + 29 = 149$ minutes. Each episode is 20 minutes long, so the number of full episodes that can be watched is the integer part of $\frac{149}{20}$, which is $\boxed{7}$.

24. First, we compute what Matt pays. His discount is $10\% \times \$20 = 0.1 \times \$20 = \$2$, so he pays \$18 total. Tanny pays $\$25 = \$20 + \$5$, so his 15% discount applies only to \$20: $15\% \times \$20 = 0.15 \times \$20 = \$3$ is his discount, so he pays in total \$22. Thus, the answer is $\boxed{\text{Tanny, \$22}}$.

25. Each block is cut into 7 slices, which means that 6 slices were made on each. 5 minutes is 300 seconds, so $\frac{300 \text{ sec}}{60 \text{ slices}} = \boxed{5}$ sec/slice.

26. If possible, we want our value to be negative, and we want it to be "as negative as possible" in that its size is maximized. Thus, we choose the "most negative" number in the set, -7, and multiply it by the greatest (i.e. "most positive") number, 3, yielding $\boxed{-21}$.

27. For each topping, Young Guy has 2 choices: to choose it or to not choose it. Since there are three toppings, the final answer is $2 \times 2 \times 2 = \boxed{8}$.

28. An isosceles triangle has two distinct side lengths; that of the legs (which are the same), and that of the base. If we are given two side lengths of an isosceles triangle and they are not equal, then the third side must be congruent to one of the two. We are told that

this triangle has side lengths 7 and 15, so the third must be 7 or 15. However, by the triangle inequality, the third side cannot be 7, because $7 + 7 < 15$; meanwhile, $15 + 15 > 7$ and $15 + 7 > 15$. Thus the only possible perimeter is $15 + 15 + 7 = \boxed{37}$.

29. The sum of the first n positive integers is given by $\frac{n(n+1)}{2}$. $10 + 11 + 12 + \cdots + 99 + 100 = 1 + 2 + 3 + \cdots + 99 + 100 - (1 + 2 + 3 + \cdots + 9 + 10) = \frac{100(101)}{2} - \frac{10(11)}{2} = 5050 - 45 = \boxed{5005}$.

30. Their path has 8 stages (each distance measured in feet):

    (a) 10 (up the first wall)
    (b) 0.5 (across the first wall)
    (c) 8 (10 – 2, down the first wall)
    (d) 25 (across)
    (e) 9 (11 – 2, up the second wall)
    (f) 1.5 (across the second wall)
    (g) 10 (11 – 1, down the second wall)
    (h) 10 (across)

    Adding gets $10 + 0.5 + 8 + 25 + 9 + 1.5 + 10 + 10 = \boxed{74}$ feet.

31. If the product of two integers is odd, then both of them must have been odd. This means that the probability of the product being odd is the same as the probability of having rolled two odd numbers. The probability of rolling an odd number for a single roll is $\frac{1}{2}$, and dice rolls are independent of each other, so we can compute the probability by multiplying: $\frac{1}{2} \times \frac{1}{2} = \boxed{\frac{1}{4}}$.

32. Alex catches up by $\frac{1}{2}$ of a foot every second. Since he needs to catch up on what is initially a 20 foot lead, he will take $\boxed{40}$ seconds to do this.

33. Note that 10 Dragons = 25 YoungGuys, and 10 Dragons = 75 Puzzle, so 25 YoungGuys = 75 Puzzle, or 1 YoungGuy = $\boxed{3}$ Puzzle.

34. A square has four equal sides, so its perimeter is four times a side length, i.e. $4 \cdot 18 = 72$. A hexagon has six sides, so each side is $\frac{1}{6}$ of the perimeter, which is same as the square's, so the side length is $\frac{72}{6} = \boxed{12}$.

35. Let x be her favorite number. The first few prime numbers are $2, 3, 5, \ldots$. So, the smallest odd prime is 3. Rewriting the question as an equation, we have that $3(x+9) = 3^4 = 81 \implies 3x + 27 = 81 \implies 3x = 54 \implies x = 18$. We are looking for half of x, which is $\boxed{9}$.

36. Suppose Kelvin's roll is written as an ordered pair, with the first number coming from the first die. Then, the rolls that obtain a 5 are $(1, 4)$, $(2, 3)$, $(2, 3)$ (duplicated because 2 appears twice on the first one), $(4, 1)$. There are 4 desired outcomes out of $6 \times 6 = 36$ total possible outcomes, for a final answer of $\frac{4}{36}$, or $\boxed{\frac{1}{9}}$.

37. First, note that $\frac{x+y}{xy} = \frac{1}{x} + \frac{1}{y}$, and the two addands are independent of each other. Thus, we want to minimize them individually. The first is minimized when x is maximized, which is at 2015; similarly, $y = 403$. The final answer is $\frac{1}{2015} + \frac{1}{403}$, or $\boxed{\frac{6}{2015}}$.

38. Let's relabel the letters slightly: $\mathrm{JHM_1M_2C}$. Since they are all different, there are $5! = 120$ arrangements. But in our original word, the M's were the same. Since in a given arrangement, there are $2! = 2$ ways to arrange them, we divide out by this factor of overcounting, to get $\boxed{60}$ arrangements.

39. Since we begin at noon, the 6 lines of code for that day are already written. Therefore, 3 days later, or on the deadline, KM will have $30 + 3 \cdot 6 = 48$ lines of code done. The lines of code at day x past the deadline would be $48 + 6x$. Since KM will only submit when he has a multiple of 17 lines code, the problem reduces down to solving the modular equation $48 + 6x \equiv 0 \ (mod\, 17)$. Then,

$$48 + 6x \equiv 6x - 3 \equiv 2x - 1 \equiv 0 \ (mod\, 17).$$

Therefore $x = 9$, so KM will submit his code $\boxed{9}$ days late. (Don't be like KM! Dr. Nevard will not like you.)

40. Points $(4,\ 4)$, $(5,\ 3)$, $(6,\ 2)$ are collinear, so the point $(5,\ 3)$ is negligible. We proceed by drawing a rectangle with coordinates $(1,\ 1)$, $(1,\ 4)$, $(6,\ 1)$, and $(6,\ 4)$, and subtracting out the areas not in the polygon. The area of the rectangle is $3 \cdot 5 = 15$; the bottom right triangle has area $1 \cdot 5 \div 2 = 2.5$; the area of the top right triangle is $2 \cdot 2 \div 2 = 2$, and finally, the top left figure has area $1 + 2 \cdot 1 \cdot 2 = 3$. Therefore, the area of the polygon then is $15 - 3 - 2 - 2.5 = \boxed{7.5}$

41. The important trend here to notice is that the number of pencils Arkun has decreases throughout the week, but only increases at the end rather than gradually. Since yesterday he found 4 pencils, the 5 day period starts today. At the start of each week, Arkun must have at least 5 pencils, since over the course of the first four day he will lose 4. In 22 days, there will be 4 complete 5 day periods, and the first 2 days of the next one. Since there are only 2 pencils lost, the start of the last period does not require Arkun to have 5 pencils. However, all previous periods do, namely the 4th one. Since during each complete cycle Arkun loses 1 pencil, at the start Arkun must have $3 + 5 = \boxed{8}$ pencils.

42. Let's look at the numbers that have remainder 4 when divided by 7: $4, 11, 18, 25, 32, 39, 46, \ldots$. The numbers with remainder 2 when divided by 5 are $2, 7, 12, 17, 22, 27, 32, 37, 42, 47, \ldots$. These lists first overlap at $\boxed{32}$ chairs.

43. Alex travels the first leg of his trip in $\frac{300}{60} = 5$ hours, the second in $\frac{90}{90} = 1$ hour, and the third in $\frac{360}{40} = 9$ hours. The total length of time is $5 + 1 + 9 = 15$ hours. The total distance is $300 + 90 + 360 = 750$ hours. Thus, his average speed is $\frac{750}{15} = \boxed{50}$ miles per hour.

44. The answer, $\boxed{12}$, can be reached by correctly graphing and counting. Things to keep in mind are that there are two parts to the first graph, intersections, and that x axis and y axis are not part of the graph.

45. The only way for no two of the three to not have birthdays on the same day of the week is if they all have birthdays on different days. Considering the people one at a time, regardless of the day of week that the first person's birthday falls on, the chance that the second person has their birthday is not on the same day of the week is $\frac{6}{7}$. If this occurs, the chance that the third person's birthday does not fall on either of the days that the first or second person's distinct days of the week is $1 - \frac{2}{7} = \frac{5}{7}$. Since these events are independent, the total probability is $\frac{6}{7} \times \frac{5}{7} = \frac{30}{49}$. We are looking for chance this doesn't happen: $1 - \frac{30}{49}$, or $\boxed{\frac{19}{49}}$.

46. Column sum = sum of numbers in grid = 45; row sum = sum of numbers in grid = 45; diagonal sum = sum of odd numbers in grid + middle number. Call that middle number n. $45 + 45 + (1 + 3 + 5 + 7 + 9) + n = 115 + n = 122$; $n = \boxed{7}$.

47. Suppose Jon is telling the truth. Then, his statement is not consistent with Alex's statement, so one of them must be lying. If Alex is lying, then there are no Liars, which is clearly false, because Alex would be a Liar! So Jon is a Liar and Alex is a Truth-teller. This validates Soonho's statement, since 5 total Liars is now no longer possible. Mike and Claire cannot both be Liars, and cannot both be Truth-tellers, so exactly one must be lying. This means that the Liars consist exclusively of Jon and either Mike or Claire; a total of $\boxed{2}$.

48. Write out the sum. Start with $a = 1$: $1 \times 1 + 1 \times 2 + \cdots + 1 \times 9 = 1(1 + 2 + \cdots + 9)$. For $a = 2$, the sum is $2(1 + 2 + \cdots + 9)$. By the distributive property, the final answer is $(1 + 2 + \cdots + 9)(1 + 2 + \cdots + 9) = (45)(45) = \boxed{2025}$.

49. Since there is only one way to arrange of A, B, and C, namely A, B, C, the number of ways to arrange them is $\binom{6}{3} = 20$. Now we are left with 2 choices to arrange the rest 3 letters, D, E, F or D, F, E. Therefore, our final answer is $20 \cdot 2 = 40$.

50. Suppose the questions are numbered 1,2,3,4,5,6,7, and suppose Alex answers numbers 1 and 2 correctly. Now, suppose Ryan answers **none** correctly in common with Alex; this is the opposite of what we are asked to find. The probability of this is the number of ways that Ryan can answer 3 correctly out of 3,4,5,6,7 divided by the number of ways that Ryan can answer 3 correctly out of 1,2,3,4,5,6,7; i.e., $\frac{\binom{5}{3}}{\binom{7}{3}} = \frac{10}{35} = \frac{2}{7}$. The complement of this eventuality is what we want: $\boxed{\frac{5}{7}}$.