2015 Joe Holbrook Memorial Math Competition
5th Grade Exam

The Bergen County Academies Math Team

October 11th, 2015

Instructions

DO NOT TURN OVER THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

The Joe Holbrook Memorial Math Competition is a 90-minute, 50-question exam. Each question has exactly one correct answer; writing two different answers, EVEN IF ONE IS CORRECT, is worth no credit. In addition, **only answers written on the answer sheet provided will be graded.**

Be advised that proctors cannot answer any questions about terminology, notation or the questions.

On the JHMMC, you may only use writing utensils, erasers, and scrap paper provided by the proctors. You MAY NOT use calculators, compasses, protractors, straightedges, or your own scrap paper.

All answers must be fully simplified and exact. For example:

- Write $2\sqrt{2}$, rather than $2.83\ldots$
- Write $\frac{4}{3}$, rather than $1\frac{1}{3}$ or $1.33\ldots$

In addition, know that $[A_1A_2\cdots A_n]$ denotes the area of n-gon $A_1A_2\cdots A_n$.

1. What is $11 + 10 + 9 + 7 + 5 + 4 + 2 + 1 + 1$?
2. AJ drank two water bottles in the morning, three water bottles at lunch, and five water bottles during dinner. How many water bottles did he drink total?
3. Moonbeam is sixteen years old, while Sunshine is twenty. Of the two, who was born second?
4. A textbook weighs 5 pounds. While holding the textbook, Thomas weighs 350 pounds. How much does Thomas weigh without the textbook?
5. Nemo has 11 goldfish in his fish tank. Each goldfish needs to eat 19 goldfish flakes daily. How many goldfish flakes, in total, does Nemo need each day to be able to feed all 11 goldfish?
6. What is $14 + 4 \times \frac{15}{3} - 9$?
7. What is the remainder when 123456 is divided by 5?
8. How many times does the Sun set between noon on Saturday and noon on the following Thursday?
9. Find the 100th member of the following sequence: $1, 4, 7, 10, \ldots$
10. Compute $3(3(3 + 2) + 2(3 + 2)) + 2(3(3 + 2) + 2(3 + 2))$.
11. Songpai decides to plant bonsai trees in the front of his lawn. He measures and finds out that his lawn measures 30 meters. He wants to plant a bonsai tree every 5 meters, and he wants to plant as many as possible. How many bonsai trees will he need?
12. What is the sum of the number of faces and edges of a pyramid with an octogonal base?
13. Kelvin the Frog is having trouble finding his way around college. He knows that the cafeteria is 20 hops away from his dorm, while the gym is 10 hops away. What is the difference, in hops, between the longest possible distance from the cafeteria to the gym and the shortest possible distance?
14. What is $0 - 1 + 2 - 3 + 4 - 5 + \cdots - 99 + 100$?
15. The mean of 2015 consecutive integers is 2. What is the median?
16. How many multiples of 17 are three-digit positive integers?
17. Sung Hyup was fifteen years old four years ago. Sung Hyun is three years younger than him. How old will Sung Hyun be in two years?
18. The New Horizons probe reached Pluto on July 14, 2015. If it was launched from Earth on January 19, 2006, how many full calendar months did it spend in space before reaching Pluto?
19. After returning home from school, Zack wants to watch his favorite TV show, the Adventures of Super Matthew. If each episode is twenty minutes long, and Zack gets home at 5:00, how many complete episodes can he watch before he must begin his homework at 7:29?
20. Jungle Jim, the proprietor of the jungle gym, is charging admission. If the admission fee for Matt is \$20, but he has a coupon for a 10% discount, and the admission fee for Tanny is \$25, but he has a coupon for a 15% discount on every dollar after \$5 that he pays, who pays more, and how much does he pay?

21. Jungle Jim is now actively preventing people from entering the jungle gym! To enter, Zack and Erik now have to scale and descend two completely vertical walls in succession. They start at sea level, and climb over a wall that is 10 feet high and 0.5 feet thick, which leads to an area of elevation 2 feet. They go forward 25 feet, and climb over a wall that is 11 feet high (relative to sea level) and 1.5 feet thick. This leads to an area which is at elevation 1 foot above sea level. They go forward 10 feet, and have reached the jungle gym! What is the total length of their path, in feet?

22. What is the least product one could obtain by multiplying two numbers in the set $\{-7, -5, -1, 1, 3\}$?

23. Alex the Kat is chasing Thomas the Mouse, who has a 20 foot lead. Mice run at 1 foot every two seconds, whereas Alex the Kat runs at 1 foot every one second. How many seconds does it take for Alex the Kat to catch Thomas the Mouse?

24. Compute $(23 + 46 \times 23 + \frac{46}{2}) - 48 \times 23$.

25. A square and a regular hexagon have the same perimeter. The square has side length 18. What is the side length of the hexagon?

26. Jen, Josh, and Joonyoung eat $\frac{2}{5}$, $\frac{1}{10}$ and $\frac{1}{4}$ of a 20-ounce cake, respectively. After they finish eating, they give the remainder of the cake to Max, who then eats the rest. How many more ounces of cake did Jen eat than Max?

27. What is the last digit of 2^{245}?

28. What is the smallest positive integer divisible by 4, 5, and 6?

29. *Double nine* dominoes are composed of 2 numbers that range from 0 to 9 and can be rotated–i.e., the domino labeled with 1 and 2 is the same as one labeled with 2 and 1. If **The Stangulator** has a complete set with no repeats, how many *double nine* dominoes does he have?

30. When I add 9 to my favorite number and triple the result, I get the fourth power of the smallest odd prime number. What is half of my favorite number?

31. How many distinct arrangements are there of the letters JHMMC?

32. Define a sequence of integers as follows: $a_0 = 0, a_1 = 1, a_2 = 2$, and for $n \geq 3$, $a_n = a_0 + \cdots + a_{n-1}$. Find the value of a_8.

33. The 400-digit number $1234567812345678 \ldots 12345678$ is written on a piece of paper. June then repeatedly erases every 7th digit of the number. When he runs out of digits to erase, he begins this erasing process over again and again, starting from the beginning of the remaining number. At the end of this process, a six-digit number remains. What is this number?

34. A cube has side length 3. Through each pair of faces, a square is drilled through the center of each face, with side length 1, and such that the sides of the hole are parallel to the sides of the faces. What is the new volume of the solid?

35. What is the probability that a random two digit integer is greater than or equal to the number resulting from switching its digits?

36. Ryan the $\mathfrak{A}$ randomly draws two points on the circumference of a circle. What is the probability that the points lie within 60° of one another?

37. If Arkun Lars loses 1 pencil at the end of each day but finds 4 pencils at once at the end of every 5 days, how many pencils does he need to have at the start to have a pencil every day for 22 days?

38. Alex the Kat is going on a road trip after his retirement. He travels the first 300 miles at 60 miles per hour, the next 90 miles at 90 miles per hour, and the last 360 miles at 40 miles per hour. What is his average speed throughout the trip?

39. Let $a = 3^{2015}, b = 2015^3, c = 1009^{1009}$. Using their variable names, order the three numbers from largest to smallest.

40. The real numbers $a-7$, $b+4$, and $c+6$ form a geometric sequence, and numbers a, b and c lie in an arithmetic sequence and have an arithmetic mean of 46. What is the arithmetic mean of the terms in the geometric sequence?

41. The graphs of $y = \frac{1}{x}$, $y = 1$, $y = x$ and $y = x^2$ cut the plane into how many pieces?

42. There is a sequence of positive real numbers $a_1, a_2, a_3, \ldots$ such that for $n \geq 2$, each term a_n is the product of all the previous terms. If $a_7 = 41$, what is a_8?

43. Young Guy seems to have caught himself in quite a predicament! Up at the board in his math class, he has been asked by his teacher, Dr. Lal, to compute the product of two numbers a and b, not necessarily distinct, each between 1 and 9, inclusive. But alas, Young Guy has forgotten which two numbers he was assigned, and decides to choose a random pair of numbers and carry out the multiplication. What is the sum of all possible products $a \times b$ Young Guy can reach?

44. How many positive integer factors does $2^{2^4} - 1$ have?

45. If there is a point P inside rectangle $ABCD$ such that $AP = 20$, $BP = 24$, and $CP = 15$, what is DP?

46. If $x^4 - 4x^3 + 2x^2 - 4x + 1 = 0$ and $y = |x - \frac{1}{x}|$, what is the value of y?

47. The sum of the first two terms of an infinite geometric sequence is $\frac{8}{3}$, and the sum of all the terms is 3. Find the sum of all possible first terms of the sequence.

48. Mikako places 10 lines on the plane so that there are 34 intersection points, each of which has two lines going through it. Two lines are in the same set A_i if and only if they are parallel. What is the maximum value of $|A_1| \times |A_2| \times \cdots \times |A_n|$?

49. Suppose a and b are the roots of the quadratic $P(x) = x^2 - 52x + 365$. Let $Q(x) = x^2 - (ab)x + (a+b)$, and let c and d be solutions to the equation $Q(x) = 0$. Find $a+b+c+d$.

50. Rebecca can only move parallel to the x-axis (at a constant speed of 20 mph) and parallel to the y-axis (at a constant speed of 10 mph), while The Great Bustard can move in any direction at 10 mph. If both Rebecca and The Great Bustard start at the origin, what is the area of the region $|x| < 1$ and $|y| < 1$ in the xy plane that The Great Bustard can reach before Rebecca?