Joe Holbrook Memorial Math Competition

4th Grade Solutions

October 17, 2021

1. $2021 - 2120 = -99$. $-99 + 1202 = \boxed{1103}$.

2. By the PEMDAS order of operations: $(7+7) \times ((7 \div 7) \div 7) + (7-7)^7 = 14 \times (1 \div 7) + 0^7 = 14 \times \frac{1}{7} + 0 = \boxed{2}$.

3. If the square has a side length of s, then its perimeter is $4s$. So, if $4s = 40$, then $s = 10$, and the area of the square is $s \cdot s = 10 \cdot 10 = \boxed{100}$.

4. Let x be the number of windows on her Mac. Then she has $3x$ windows on her PC. In total, she has $x + 3x = 4x$ computers, so Bianca has $4x = 20 \implies x = 5 \implies 3x = \boxed{15}$ windows on her PC.

5. There are $90 - 45 - 30 = 15$ blue beads, so the ratio of blue beads to red beads is $15 : 45 = 1 : 3 \implies a + b = \boxed{4}$.

6. After the 34 dalmatians go missing, there are $101 - 34 = 67$ dalmatians left. These can be split into 5 groups of 13 dalmatians with $\boxed{2}$ leftover.

7. In each subsequent day, Catherine drives four more miles. The value in each row increases by 4:

   Day 1: 12 miles
   Day 2: 16 miles
   Day 3: 20 miles
   Day 4: 24 miles
   Day 5: 28 miles

   Therefore, on the 5th day, Catherine will drive $\boxed{28}$ miles.

8. We can calculate $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} \cdot 100 = \frac{1}{5} \cdot 100 = \boxed{20}$ cents by cancelling.

9. Each second, Ashley catches up $13 - 3 = 10$ meters. Thus, it will take $\frac{50}{10} = \boxed{5}$ seconds for Ashley to catch up 50 meters.

10. After he drinks x ounces of water, Erez has $32 - x$ ounces left. Then he drinks $\frac{1}{3}$ of this remaining water, and thus has

$$\frac{2}{3}(32 - x) = \frac{64}{3} - \frac{2}{3}x = 16$$

    ounces left. Multiplying this equation by 3 to simplify it, we get

$$64 - 2x = 48.$$

    Then solving for x yields $x = \boxed{8}$.

11. The side length must be $x^2 = 16$, so $x = 4$. Thus, the pentagon has 5 sides of this length, so the perimeter is $5 \cdot 4 = \boxed{20}$.

12. Since there are 20 students, it will take $20 \cdot 5 = 100$ seconds to complete the actual pencil sharpening. Additionally, there are 19 exchanges between students that last 2 seconds each, for a total exchange time of $19 \cdot 2 = 38$ seconds. Thus, the total time it would take is $100 + 38 = \boxed{138}$ seconds.

13. Pineapple's iPine charges 15% per hour, which means it charges 1% every 4 minutes. Therefore, it will charge 100% in 400 minutes. Jamsung's Jalaxy charges 1% every 6 minutes, so it will charge to 100% in 600 minutes. Thus, the difference in minutes will be $600 - 400 = \boxed{200}$ minutes.

14. We can add probabilities to find the probability that Yul arrives by a certain time. There is a 20% chance she arrives by 5pm, a $20 + 20 = 40\%$ chance she arrives by 5:10pm, and $40 + 30 = 70\%$ chance she arrives by 5:20pm. Thus, Alicia should arrive at 5:20pm, which is $\boxed{20}$ minutes late.

15. In essence, we are trying to find the diameter of the circular park, as they are walking the path created from the opposite ends of the circle. Because both friends walked 18π miles, they walked 36π miles in total, implying that the circumference of the circle is 36π. The circumference of a circle is equal to its diameter times π, so the diameter is is $\boxed{36}$.

16. Tommy makes 3 shots out of 12, so his success rate is $\frac{3}{12} = 25\%$. Tommy has a success rate that is triple of Tommy's, so he succeeds 75% of the time. This means he makes $16 \cdot \frac{3}{4} = \boxed{12}$ shots.

17. First, we consider performances that are hilarious. This implies every judge rated the performance either a 7, 8, 9, or 10. There are three judges, so the lower bound for a hilarious performance is a total score of $7 \cdot 3 = 21$, whereas the upper bound is $10 \cdot 3 = 30$. There are $30 - 21 + 1 = 10$ possible scores for hilarious performances.

    For a performance to be embarrassing, each judge must have rated it either a 1, 2, or 3. The lower bound here is a total score of $1 \cdot 3 = 3$, whereas the upper bound is $3 \cdot 3 = 9$. There are $9 - 3 + 1 = 7$ possible scores for embarrassing performances.

    (Note that all the scores in the ranges are attainable because the total score increases by 1 each time a judge increases his or her score by 1.)

    In total, there are $10 + 7 = \boxed{17}$ possible scores for performances that are either hilarious or embarrassing.

18. If the answer is a one-digit number, then the answer would be $1 + 11 = 12$, which is not a one-digit number. If the answer is a two-digit number, the answer would be $2 + 11 = 13$, which is indeed a two-digit number. Thus, the answer is $\boxed{13}$.

19. The total area of the cookie is $\pi \cdot 10^2 = 100\pi$. Since a quarter of the area is covered by chocolate chips, the total area of the chips must be 25π. The area of each chip is $\pi/cdot1^2 = \pi$, so there are $\boxed{25}$ chips on the cookie.

20. We start by finding L. Lili's list is $6, 12, \ldots, 96$ which can be rewritten as $1 \cdot 6, 2 \cdot 6, \ldots 16 \cdot 6$. Clearly, $L = 16$. Zoey's list is $1, 5, \ldots, 97$ which can be rewritten as $0 \cdot 4 + 1, 1 \cdot 4 + 1, ..., 24 \cdot 4 + 1$. Clearly $Z = 25$. Hence our answer is $16 - 25 = \boxed{-9}$.

21. Let $AE = x$. EB therefore equals $5 - x$. The area of $\triangle AED = \frac{1}{2} \cdot 3x = \frac{3x}{2}$. The area of $\triangle BEC = \frac{1}{2} \cdot 3 \cdot (5 - x) = \frac{15 - 3x}{2}$. The sum of these areas is equal to $\frac{15}{2}$, and since the area of rectangle $ABCD$ is 15, the area of $\triangle DEC$ is $15 - \frac{15}{2} = \frac{15}{2}$, so our final answer is $15 + 2 = \boxed{17}$.

22. Note that the prime factorization of 25 is $5 \cdot 5$, so the only integers that divide it evenly are 1, 5, -1, and -5. Indeed, $1 \times 5 \times -1 \times -5 = 25$, and $1^2 + 5^2 + (-1)^2 + (-5)^2 = 1 + 25 + 1 + 25 = \boxed{52}$.

23. We can simply test the first few primes. We can see that 2, 3, and 5 don't work, whereas $\boxed{7}$ does (6 and 8 both have 4 divisors).

24. Since the area of one lilypad is π square feet, the total area that the lilypads cover is 108π square feet. Thus, because Kelvin the Frog's home has total area of $6 \cdot 8$ square yards, or $48 \cdot 9$ square feet, the uncovered area is $432 - 108\pi$ square feet. So, the answer is $432 + 108 = \boxed{540}$.

25. The only positive integers that have an odd number of divisors are perfect squares. Since Rosie's favorite number is less than 100, this gives us 9 possible numbers: $1, 4, 9, 16, 25, 36, 49, 64, 81$. We see that $\boxed{25}$ must be Rosie's favorite number because it is the only number in this set that has a units digit of 5. All the other numbers do not have a unique units digit, so Jennie would not have been able to figure out Rosie's favorite number.

26. To make the numbers cleaner, let the potato farm have an area of 3400. A clone of Jaiden can harvest 2 units per day while a clone of Lance can harvest 1 unit per day so we get the equation $2x + y = 3400$ for $x, y, \geq 0$. This has $\boxed{1701}$ solutions, since x can take any integer value from 0 to 1700.

27. Since Cookie Monster's cookie costs 67 cents, we know he must use the 2 pennies to pay. Also, because we have the condition that he must pay using at least 3 dimes, we can subtract that out so that we do not have any more restrictions left. Now, we need to find the number of ways to pay for 35 cents using at max 3 pennies, 5 nickels, 2 dimes, and 5 quarters. This can be done with 2 nickels + 1 quarter, or 1 dime + 1 quarter, 3 nickels + 2 dimes, or 5 nickels + 1 dimes. This yields the answer of $\boxed{4}$.

28. In order to guarantee that Harry has picked out a pair of red socks, Harry must consider the worst possible scenario, which is that Harry picks out every blue sock and every green sock before he picks even a single red sock. If Harry picks every green sock and every blue sock, he would only have red socks to choose from after that. So, he would only have to pick 2 more socks to ensure that he has picked at least one pair of red socks. In total, this would be $12 + 15 + 2 = \boxed{29}$ socks.

29. We can connect the center of the rectangle (also the center of the circle) to a vertex of the rectangle (which lies on the circle) to get that half the diagonal of the rectangle is equal to 20, the radius of the circle. From here, we can use the Pythagorean Theorem to get that half the other side of the rectangle is equal to 12, so the entire rectangle has dimensions 32×24 which yields 768 square miles. The original circle was 400π square miles, so $n + m = 400 + 768 = \boxed{1168}$.

30. Call Maui's distance to the shore x, and his distance to the cave y. Additionally, call his running speed s_r and his flying speed s_f. Since time is equal to distance divided by speed, $\frac{x}{s_r} = \frac{y}{s_r} + \frac{x+y}{s_f}$. Since we know $s_f = 4s_r$, this equation can be rewritten as $\frac{x}{s_r} = \frac{y}{s_r} + \frac{x+y}{4s_r}$. Finally we can solve the equation:

$$\begin{aligned} \frac{x}{s_r} &= \frac{y}{s_r} + \frac{x+y}{4s_r} \\ 4x &= 4y + x + y \\ 3x &= 5y \\ \implies x : y &= 5 : 3 \end{aligned}$$

Therefore, $a = 5$ and $b = 3$, so $a + b = \boxed{8}$.

31. Since we want the number to be as small as possible, a must have the smallest possible value, so let $a = 1$. Then the next smallest number is 2, so let $b = 2$. The only possible digit smaller than two is now 0, so $c = 0$. Then, we take the two smallest remaining numbers to assign to d, e. Since we know $d > e$, $d = 4$ and $e = 3$. We get a result of $\boxed{12043}$.

32. Since the corresponding side lengths of the two triangles have the same ratio, the ratio of their perimeters must also equal this ratio of corresponding sides. This unknown ratio can be 3 values depending on which side of the first triangle corresponds with the 12 cm side of the second triangle:

$$\begin{aligned} \frac{2}{12} &= \frac{1}{6} \\ \frac{4}{12} &= \frac{1}{3} \\ \frac{3}{12} &= \frac{1}{4}. \end{aligned}$$

We know that this ratio must equal $\frac{2+4+3}{P} = \frac{9}{P}$ where P is the perimeter of the second triangle. Using the three possible ratios found before, we find $P = 54, 27$, or 36. Hence, our answer is $54+27+36 = \boxed{117}$.

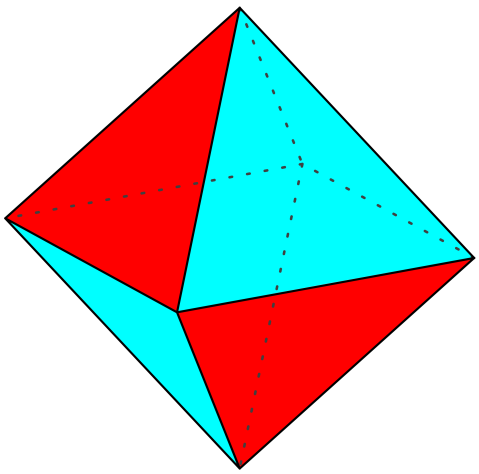
33. This can be written as $(3 \cdot 2 \cdot 1) \cdot (5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) \cdot (8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) = 2^{11} \cdot 3^4 \cdot 5^2 \cdot 7$. Then a perfect cube that divides this must be in the form of $2^a \cdot 3^b \cdot 5^c \cdot 7^d$ where a, b, c, d must be multiples of 3, i.e.

$$a \in \{0, 3, 6, 9\}, b \in \{0, 3\}, c \in \{0\}, d \in \{0\}.$$

Thus, there are $4 \cdot 2 \cdot 1 \cdot 1 = \boxed{8}$ perfect cubes that divide the given expression.

34. There are $\binom{9}{2} = \frac{9 \cdot 8}{2} = 36$ ways to place the green books, and of the remaining 7 spots there are $\binom{7}{3} = \frac{7 \cdot 6 \cdot 5}{3!} = 35$ ways to place the blue books, for a total of $36 \cdot 35 = \boxed{1260}$ possibilities.

35. The answer is $\boxed{2}$. Just alternate colors on the top, and do a corresponding alternating pattern on the bottom.



36. First, write the first 21 positive integers. The numbers $(1, 2, 4, 8, 16)$ are all connected and have 2 different colorings: R, B, R, B, R or B, R, B, R, B. The same is true for the groups $(3, 6, 12), (5, 10, 20), (7, 14)$, and $(9, 18)$. The only numbers that are independent are 11, 13, 15, 17, 19, and 21 and they each have two choices R or B. Lastly, doing the calculation gives $2^5 \cdot 2^6 = \boxed{2048}$ different colorings.

37. Choose F on AB such that EF is perpendicular to AB, then extend EF to meet CD at G. Notice $\triangle EDG$ is a right triangle with hypotenuse DE. We know $DG = \frac{1}{2}, FG = 1, EF = \frac{1}{2}$. (The last one might be harder to see, but $\triangle AEF$ and $\triangle BEF$ are both also isosceles right triangles.) By the Pythagorean Theorem, $DE = \sqrt{(\frac{1}{2})^2 + (1 + \frac{1}{2})^2} = \sqrt{\frac{10}{4}} = \sqrt{\frac{5}{2}} \implies a + b = 5 + 2 = \boxed{7}$.

38. Consider the cases of how many consecutive matches Nikhil wins. If he wins only 3, the possible sequences are $WWWL*, LWWWL$, or $*LWWW$ where $*$ denotes either a win or a loss. This results in $\frac{2}{81} + \frac{4}{243} + \frac{2}{81} = \frac{16}{243}$. If Nikhil wins only 4, the possible sequences are $WWWWL$ or $LWWWW \implies \frac{2}{243} + \frac{2}{243} = \frac{4}{243}$. If Nikhil wins all 5, there is only $WWWWW \implies \frac{1}{243}$. Thus, in total, Nikhil there is a $\frac{16}{243} + \frac{4}{243} + \frac{1}{243} = \frac{21}{243} = \frac{7}{81} \implies \boxed{88}$.

39. Plug in $(x - a)$ and $(y - b)$ in x and y respectively. Then we get $y - b = 3(x - a)^2 + 2(x - a) - 1$. Expanding and simplifying this, the expression becomes $y = 3x^2 - 6ax + 3a^2 + 2x - 2a - 1 + b = 3x^2 - (6a - 2)x + 3a^2 - 2a - 1 + b$. Then $6a - 2 = 10$ and $3a^2 - 2a - 1 + b = 12$. We get $a = 2$ and $b = 5 \implies 2 + 5 = \boxed{7}$.

40. Using geometric probability, a graph can be made mapping the situations where Jasmine and Aladdin's will meet each other. For any time one of them arrives, the other has a 60 minute window to show up (30 minutes before or after). The graph will look like the one below. The center part is the chance where they will meet. Out of the whole graph, that section makes up $\frac{11}{36}$ of it. So $36 - 11 = \boxed{25}$.