Joe Holbrook Memorial Math Competition

6th Grade Solutions

October 16, 2022

1. The problem allows us to set up the equation $0.24x = 0.30y$. We are asked to find what percent of x is y, so we need to isolate y. Dividing by 0.30 on both sides yields $y = \frac{0.24}{0.30}x = 0.80x$, meaning our answer is $\boxed{80}$

2. We can simply try each prime in ascending order starting from 3 - we find that $3^2 + 4$, $5^2 + 4$, and $7^2 + 4$ are prime but $\boxed{11}^2 + 4 = 125$ is not.

3. For them to both make 1200 cutouts, Timmy takes $1200/10 = 120$ hours to finish, and Shawn takes $1200/15 = 80$. The answer is $\boxed{40}$.

4. The most effective way to cut the cake is to start at a corner and cut each subsequent piece right next to the previous cut. This minimizes the number of leftover pieces of cake that are too small to be turned into a 2" x 2" piece. However, notice that following this method, there will always be a 1" x 4" strip leftover which cannot be turned into any pieces. Therefore, only the 12" x 4" portion is usable, giving us $\frac{12 \cdot 4}{2 \cdot 2} = \boxed{12}$ pieces.

5. Because a problem's difficulty is equal to the number of minutes it takes to write a solution for it, we can deduce that Lance's problems' average time to write solutions for is 6 minutes. So it'll take $12 \times 6 = \boxed{72}$ minutes for him to write all of his solutions.

6. $293 \div 14 = 20$ R 13, so Eshaan needs $20 + 1 = \boxed{21}$ days to listen to at least 293 songs.

7. Rose should apply the 20% off coupon first because the base value it is being applied to is greater, meaning a greater value is being deducted. This coupon reduces the price to $100 \cdot 0.8 = 80$. After the second coupon, Rose only has to pay $80 - 10 = \boxed{70}$.

8. Because the nobleman was able to figure out the king's number after the questions, only one number satisfies the king's 3 answers. Let's analyze those possible answers and the numbers that satisfy those answers. (Y indicates yes, N indicates no, the responses are given in the same order as the questions)

   - YYY: 3, 5, 7 satisfy this
   - YYN: 2
   - YNY: 1, 9
   - YNN: 4, 6, 8
   - NYY: 11, 13, ...
   - NYN: no such number satisfies this
   - NNY: 15, 21, ...
   - NNN: 12, 14, ...

   The YYN answer is the only answer that gives one possible number. Hence, this must have been the king's response and $\boxed{2}$ is the king's favorite natural number.

9. This problem is asking for an arithmetic series. The sequence is

   $$5, (5 + 4 \times 1), (5 + 4 \times 2), (5 + 4 \times 3), \ldots (5 + 4 \times 10)$$

   To find $5 + (5 + 4 \times 1) + (5 + 4 \times 2) + (5 + 4 \times 3) \ldots (5 + 4 \times 10)$, notice that it is equal to $(5 \times 11) + (4 \times (1 + 2 + 3 + \ldots 10)) = 55 + 4 \times \frac{10(11)}{2} = 55 + 4 \times 55 = \boxed{275}$ seashells.

10. If a has 9 positive multiples less than 100, then $\frac{100}{a}$ must be equal to 9 with some remainder or 10 with no remainder (since we are only counting multiples LESS than 100, not equal to 100). Using the fact that the quotient decreases as the divisor increases for positive numbers and a constant positive dividend and that $\frac{100}{9} = 11$ with remainder, $\frac{100}{10} = 10$ with no remainder, $\frac{100}{11} = 9$ with a remainder, and $100/12 = 8$ with a remainder, a can only be 10 or 11. If b has 19 positive multiples less than 100, then $\frac{100}{a}$ must be equal to 19 with some remainder or 20 with no remainder. Since $\frac{100}{4} = 25$ with no remainder, $\frac{100}{5} = 20$ with no remainder, and $\frac{100}{6} = 16$ with remainder, b can only be 5. Thus, ab is either 50 or 55. However, in either case scenario, there is only 1 multiple of ab strictly less than 100 and that is $ab \cdot 1$. Therefore, $\boxed{1}$ is our answer.

11. Solving the equation, we see that $x = -\frac{k}{3}$, and $-\frac{k}{3} < -\frac{2}{5}$ occurs exactly when $\frac{6}{5} < k$. Thus, the desired probability is $\frac{8\frac{4}{5}}{10} = \frac{22}{25} \implies \boxed{47}$.

12. This can be represented with the recurrence relation $a_n = 2 * a_{n-1} - 3$, where a_n represents the amount of cupcakes I have on day n such that $a_1 = 4$. Notice that the amount of cupcakes that I have follows the pattern that on the nth day, I will have $2^{n-1} + 3$ cupcakes. This follows from even the first day and can be more easily observed after listing terms. Thus, our answer is $2^{10-1} + 3 = \boxed{515}$.

13. It takes $\frac{450}{60} \cdot 60 = 450$ minutes to drive (distance is speed times time, and there are 60 minutes in an hour.) Similarly, it takes $\frac{400}{240} \cdot 60 + 120 = 220$ minutes to fly (the extra 120 is from the two hours of arriving/boarding), and the answer is therefore $450 - 220 = \boxed{230}$.

14. Let a be the fraction of the lawn that Alice can mow in an hour, and b be the fraction that Bob can mow in an hour. We know $a + b = \frac{1}{3}$, since they mow the entire lawn in three hours, while $a = \frac{1}{5}$ because Alice takes 5 hours to mow it. So, $b = \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$, and Bob can mow the lawn in $\frac{15}{2}$ hours, or $15 \cdot 30 = \boxed{450}$ minutes.

15. Let the number of biscuits Tasha has in total be x. According to the problem, when x is divided by 5, the remainder is 2, when x is divided by 8, the remainder is 5, and when x is divided by 7, the remainder is 4. Notice that the remainder of biscuits is always 3 less then the number of people the biscuits are being distributed among. That means the number of biscuits must be 3 less than a multiple of 5, 8, and 7. Since the least common multiple of $5, 8$, and 7 is 280, the number of biscuits in Tasha's box is $280 - 3 = \boxed{277}$ biscuits.

16. If the length of one diagonal is $2a$ and the other is $2b$, then the equations $a^2 + b^2 = 100$ and $2ab = 96$ can be made. Adding the two equations and taking the square root we get $a + b = 14$, so the sum of the diagonals is $\boxed{28}$.

17. Let x represent one side of the rectangle perpendicular to the large wall. Then our rectangle would have the sides x, x, and $300 - 2x$ (we ignore the wall as it does not contribute to our fence length). We can model the problem with a quadratic, $x(300 - 2x)$, which we want to maximize. Taking the sum of roots, the shorter side has length 75 and the other having length 150. Thus, the area enclosed is $75 \cdot 150 = \boxed{11250}$

18. We know that if a convex polygon has n sides, then its interior angle sum is given by $(n - 2) * 180°$. We also know that because it is convex, each angle is less than 180 degrees. So, we only need to find the closest multiple of 180 greater than 2022. $180 * 12 = 2160$, so our answer is $180 * 12 - 2022 = \boxed{138}$.

19. Since the area of the triangle plus the area of the pentagon is just the area of the entire square (which is $30^2 = 900$), the triangle has area $\frac{900}{3} = 300$. The area of the triangle is also $\frac{1}{2} \cdot AE \cdot AF = \frac{1}{2}^2$ by area formulas, so $AE^2 = 300 \cdot 2 = \boxed{600}$.

20. As the woman walks in a circle, the point on her hat closest to the center is $6 - 4 = 2$. That means that there will be a circle with radius 2. Using area of a circle yields a circle of alive corn that has area $2^2 \cdot \pi = 4\pi$. The point on her hat farthest from the center is $6 + 4 = 10$. This is yet another circle, this time of radius 10. We use the same formula to get, $10^2\pi = 100\pi$. We are looking for the area of dead corn, so our answer will be $100\pi - 4\pi = \boxed{96}\pi$.

21. Because $\overline{MD} \perp \overline{AC}$, $m\angle CMD = m\angle ABC = 90°$. Due to reflexivity, $m\angle ACB = m\angle ACB$. Hence, by AA similarity, $\triangle ABC \sim \triangle DMC$. We know $MC = \frac{10}{2} = 5$ and $BC = 8$, so the scale factor is $\frac{5}{8}$. Thus, $DM/AB = \frac{5}{8}$, so $DM = 6 \cdot \frac{5}{8} = \frac{15}{4}$. The area of a triangle is $\frac{bh}{2}$, so because $\triangle DMC$ is right, its area is $\frac{1}{2} \cdot (5)(\frac{15}{4}) = \frac{75}{8}$, so $m + n = \boxed{83}$.

22. Consecutive sums can be computed as the median times the number of consecutive integers. Let m, c denote the median and the number of consecutive integers. For a median that's a positive integer, there are 3 options, 6, 674 and 2022. This is because the number of consecutive integers has to be an odd number, so we can only divide 2022 by 1, 3, or 337, 1011. For a median that is not a positive integer, we need $(2m)c = 4044$ where m is odd. So, m can take $\frac{1}{2}, \frac{3}{2}, \frac{337}{2}, \frac{1011}{2}$. So, there are $\boxed{8}$ such sequences.

23. The president and vice president can be chosen 100(99) ways. From the remaining 98 people, they can be chosen 2^{98} ways. Thus the total number of possibilities is $(100)(99)(2^{98})$. The number of *ideal* possibilities are $2 \cdot (2^{98})$ because there are 2 ways to choose from Andrea and Anthony, and 2^{98} ways to choose the rest of the people. Thus, the probability is $\frac{2 \cdot (2^{98})}{(100)(99)(2^{98})} = \frac{1}{50 \cdot 99} = \frac{1}{4950}$. Thus $m+n = 1+4950 = \boxed{4951}$. □

24. We know that N's second smallest factor is p, its smallest prime factor. Thus, N's second largest factor must be $\frac{N}{p}$. So, we are saying that $\frac{N}{p} = 15p \implies N = 15p^2$. What are the possible values of p? Since p is the smallest prime factor and N is already divisible by 3 and 5, p can only be 2 or 3. So, our final answer is $15 \cdot 2^2 + 15 \cdot 3^2 = \boxed{195}$.

25. Let s be the distance between consecutive streets and a be the distance between consecutive avenues, both in feet. We know that $3s + 4a = 5280$ and $6s + a = 1420 \cdot 3 = 4260$, and wish to find $s + a$. Multiplying the first equation by two and subtracting the second equation, we get $7a = 10560 - 4260 = 6300$, so $a = 900$. We can then plug this into the second equation to get $6s = 3360$ and $s = 560$. So, our answer is $900 + 560 = \boxed{1460}$.

26. The prime factorization of $60 = 2^2 \cdot 3 \cdot 5$. Clearly, a and b can only have factors of $2, 3, 5$ in their factorizations, so suppose $a = 2^{x_1}3^{y_1}5^{z_1}$ and $b = 2^{x_2}3^{y_2}5^{z_2}$. We just need for $\max(x_1, x_2) = 2, \max(y_1, y_2) = 1, \max(z_1, z_2) = 1$. As long as one of $\{x_1, x_2\}$ is 2, the condition is satisfied, so there are $3^2 - 2^2 = 5$ ways to choose these two numbers. Using a similar process for the rest, we get that the answer is $(3^2 - 2^2)(2^2 - 1^2)(2^2 - 1^2) = \boxed{45}$.

27. The probability can be expressed as the ratio of areas of the region. We can also find the area of the complement, since when the diagram is drawn out, it ends up being a triangle with vertices $\left(\frac{3}{2}, \frac{1}{2}\right)$, $(6, 2)$ and $(0, 2)$. The area of this is just $6 \cdot \frac{3}{2} \cdot \frac{1}{2} = \frac{9}{2}$. Then, the total area of the trapezoid is $\frac{1}{2}(6 + 2) \cdot 2 = 8$. This means that the area we are looking for is $8 - \frac{9}{2} = \frac{7}{2}$, and the ratio between areas is $\frac{\frac{7}{2}}{8} = \frac{7}{16}$, giving an answer of $\boxed{23}$.

28. The number of Mayan digits needed to write a base-10 integer k is equal to the least power of 20 that is greater than k, so we want the minimum n such that $20^n > 10^{20}$. Since $20 = 10 \cdot 2$, we can rewrite the expression as $(10 \cdot 2)^n > 10^{20}$ or $2^n > 10^{20-n}$. We observe that $2^{10} > 10^3$, so if $\frac{n}{20 - n} \geq \frac{10}{3}$, then the desired inequality will be true. This occurs when $n \geq \lceil \frac{200}{13} \rceil = 16$, so any $n \geq 16$ will be sufficient.

    Meanwhile, because $65536 = 2^{16} < 10^5$, we can establish a lower bound for n: if $\frac{n}{20 - n} \leq \frac{16}{5}$ (which occurs when $n \leq \lceil \frac{320}{21} \rceil = 15$), then $2^n < 10^{20-n}$. Thus, to satisfy both inequalities, n must be greater than 15, and it can be any integer that is at least 16. Hence, the least number of Maya digits is $\boxed{16}$.

29. One way for this to be possible is if n itself contains a zero. There are 9 such numbers $(10, 20, \ldots, 90)$. Otherwise, we must arrive at one of these 9 numbers after applying p some number of times, say k times. So, if $p^k(n) = 10, 20, \ldots, 90$, we can deduce that in $p^{k-1}(n)$, one digit was 5 and the other digit was even. The possibilities are then $25, 45, 65, 85, 52, 54, 56, 58$. This yields 8 more possibilities for n. We can take it back a step further, considering $p^{k-2}(n)$ (in the case that $k \geq 2$). If we try to find two digit numbers

whose digits multiply to one of these 8 numbers, we arrive at $55, 59, 95, 69, 96, 78$, and 87, which gives 7 more possibilities for n. It is easy to see that no two digit number has digits that multiply to one of these seven, so we are done. Our final answer is $9 + 8 + 7 = \boxed{24}$.

30. Because Amir can only go to points (a, b) where at least one of $\{a, b\}$ is even, he must take two consecutive steps in a given direction in order to be able to move perpendicularly. Hence, we can consider his path as a sequence of 8 moves: 4 that consist of 2 consecutive steps right and 4 that consist of 2 consecutive steps up. These 8 moves can be arranged in any way to form a unique path: because the 4 right-steps are identical, and so are the 4 up-steps, the number of possible sequences is

$$\frac{8!}{4! \cdot 4!} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1} = \boxed{70}$$

Notice that scaling the plane down by two gives the classic "How many ways can we go from $(0, 0)$ to (n, n) with only unit moves in the up and right direction, of which the answer is $\binom{2n}{n}$.)

31. Since the boulder is tangent to the mountain and the ground, the radii of the boulder perpendicular to the ground and mountain section off a square with a side length of 5 feet. The diagonal of the square is $5\sqrt{2}$ feet long and passes through the center of Tasha's rock. That means we know that $5\sqrt{2} = 5 + r +$ the distance from the center of Tasha's rock to where the ground and mountain meet. Notice that the radii of Tasha's rock perpendicular to the ground and mountain section off a square with diagonal $r\sqrt{2}$. This means $5\sqrt{2} = 5 + r + r\sqrt{2} \implies r = \dfrac{5(\sqrt{2} - 1)}{\sqrt{2} + 1} = 5(3 - 2\sqrt{2}) = 15 - 10\sqrt{2}$. Therefore, the answer to this problem is $2(15) = \boxed{30}$.

32. The total number of possible rolls is $\dfrac{6 \cdot 6 \cdot 6}{3 \cdot 2 \cdot 1} = 36$ because there are 6 possible rolls for each die and three dice (6^3 possibilities), however, since order does not matter, the 6 ways to achieve each combination (3 roll choices for the first slot, 2 for the second roll, and 1 for the last roll) can be divided out, leaving 36 possibilities. There are 9 ways to get a roll where two of the numbers add to the third (when order does not matter). For the sake of organization, we will order the rolls in ascending order. These are

$$(1, 1, 2), (1, 2, 3), (1, 3, 4), (2, 2, 4), (1, 4, 5), (2, 3, 5), (1, 5, 6), (2, 4, 6), \text{ and } (3, 3, 6).$$

So, Elsa has a $\dfrac{9}{36} = \dfrac{1}{4}$ chance of getting a point each game. There are 8 ways to get a roll where two of the numbers multiply to the third (when order does not matter). These are

$$(1, 1, 1), (1, 2, 2), (1, 3, 3), (1, 4, 4), (2, 2, 4), (1, 5, 5), (1, 6, 6), \text{ and } (2, 3, 6).$$

So Anna has a $\dfrac{8}{36} = \dfrac{2}{9}$ chance of getting a point each game. Thus, over 36 games, it is expected that Elsa will get $\dfrac{1}{4} \cdot 144 = 36$ points and Anna will get $\dfrac{2}{9} \cdot 144 = 32$ points. This is a $36 - 32 = \boxed{4}$ point expected difference.

33. Let the probability of him typing any letter besides K, A, I, be x. This implies that the chance of typing K, A, I are each $2x$. Thus, we can write $3(2x) + 9(x) = 1$, so $x = \dfrac{1}{15}$. The chance that he types his name correctly would then be $\dfrac{2}{15} \cdot \dfrac{2}{15} \cdot \dfrac{2}{15}$. Now, to find the denominator, or the situation in which he types at least one of the letters in its correct position, we can use complementary counting, in which we determine the probability that he gets none of the letters correctly. This would be $(\dfrac{13}{15})^3$. Subtracting from 1, we have this is just $1 - \dfrac{13^3}{15^3}$. Thus, the probability is

$$\frac{\frac{2^3}{15^3}}{1 - \frac{13^3}{15^3}} = \frac{2^3}{15^3 - 13^3}$$

which is $\dfrac{4}{589}$. This would be $4 + 589 = \boxed{593}$.

34. There's only 1 way the final can be played: between Players 1 and 2. From Players 3 and 4, there are 2 ways to arrange the semifinals. There's 4! ways to decide who Players $5 - 8$ will play in the quarter finals. The same reasoning extends to find $N = 2 \cdot 4! \cdot 8! \cdot 16!$. which gives $1 + (2 + 1) + (4 + 2 + 1) + (8 + 4 + 2 + 1) = 1 + 3 + 7 + 15 = \boxed{26}$ by Legendre's formula.

35. The prime factorization of 2023 is $7 \cdot 17^2$, which can be found through applying the divisibility rule by 7 or trial division. Thus, if $2023 \mid n^2$, then n^2 is divisible by both 7 and 17^2, so n is divisible by 7 and 17. However, because $2023 \nmid n$, n cannot be divisible by both 7 and 17^2. Hence, $7 \mid n, 17 \mid n$, and $17^2 \nmid n$, so n is a multiple of 119 and so is $n - 2023k$ for any k. Thus, all remainders will be divisible by 119. There are no other restrictions on n, so because 17 is prime, the remainder can be any positive multiple of 119 less than 2023: $119(1), 119(2), \ldots, 119(16)$. These integers have sum $\frac{119 \cdot 16 \cdot 17}{2} = 2023 \cdot 8 = \boxed{16184}$.

 (Note that because $2023 \mid n$ corresponds to n having a remainder of 0 when divided by 2023, the condition that n is not divisible by 2023 does not change the sum of remainders.)

36. Because $O_1O_2 = 6\sqrt{3} < 2(6)$, the circles intersect at two points, which we will call I_1 and I_2. Denote the midpoint of $\overline{I_1I_2}$ as M. Because $O_1M + MO_2 = O_1O_2$, by symmetry $O_1M = MO_2 = 3\sqrt{3}$. We observe that the segments from each center to intersection point is a radius of a circle, so $O_1I_1, O_1I_2, O_2I_1, O_2I_2 = 6$. $3\sqrt{3} = \frac{6}{\sqrt{3}}$, so $\triangle MO_1I_1, \triangle MO_1I_2, \triangle MO_2I_1$, and $\triangle MO_2I_2$ are congruent $30-60-90$ right triangles. Hence, $I_1M = I_2M = 3$, so the area of each triangle is $(\frac{9}{2}) \cdot \sqrt{3}$. Furthermore, $m\angle I_1O_1I_2 = m\angle I_1O_2I_2 = 60^\circ$, so because the total area of each circle is 36π, the areas of the sectors enclosed by arcs $I_1O_1I_2, I_1O_1I_2$ are both 6π. Again using the symmetry of the desired region, we see that the area of the circles' union can be represented as $2(6\pi - 2[\triangle MO_2I_1])$, which is $2(6\pi - 9\sqrt{3}) = 12\pi - 18\sqrt{3}$. Hence, $a+b+c = 12-18+3 = \boxed{-3}$.

37. We can rewrite the expression as

$$\frac{\lfloor\sqrt[5]{1}\rfloor}{\lfloor\sqrt[5]{2}\rfloor} \cdot \frac{\lfloor\sqrt[5]{3}\rfloor}{\lfloor\sqrt[5]{4}\rfloor} \cdot \frac{\lfloor\sqrt[5]{5}\rfloor}{\lfloor\sqrt[5]{6}\rfloor} \cdots \frac{\lfloor\sqrt[5]{2021}\rfloor}{\lfloor\sqrt[5]{2022}\rfloor}.$$

 Note that each of these terms will cancel out except for the even perfect 5th powers, at which the denominator will be greater than 1. At odd perfect 5th powers, the denominator will be above the floor and thus it will still cancel. Thus, we can write the expression as

$$\frac{\lfloor\sqrt[5]{31}\rfloor}{\lfloor\sqrt[5]{32}\rfloor} \cdot \frac{\lfloor\sqrt[5]{1023}\rfloor}{\lfloor\sqrt[5]{1024}\rfloor} = \frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}.$$

 Thus, the answer will be $3 + 8 = \boxed{11}$

38. We can split our desired sum into $N = \sqrt{r} + \sqrt{s}$ and $K = \frac{1}{r} + \frac{1}{s}$. Let us first focus on $N = \sqrt{r} + \sqrt{s}$. If we square both sides of this equation, we get $N^2 = r + 2 \cdot \sqrt{rs} + s$. By Vieta's Formulas, we know that the sum of the solutions to the quadratic is $-(\frac{-35}{1}) = 35$ and their product is $\frac{49}{1} = 49$. Hence, $r + s = 35$, and $rs = 49$. Plugging this into our equation, we get $N^2 = 35 + 2 * \sqrt{49} = 35 + 2 * 7 = 49$, so $N = \pm 7$. However, we are told that r and s are positive real numbers, so $N = \sqrt{r} + \sqrt{s}$ must be positive, meaning that $N = 7$. Now, we can calculate $K = \frac{1}{r} + \frac{1}{s}$. This expression can be rewritten with a common denominator of rs as $K = \frac{s}{rs} + \frac{r}{rs} = \frac{r+s}{rs} = \frac{35}{49} = \frac{5}{7}$. Finally, we can add the values of N and K together: $N + K = 7 + \frac{5}{7} = \frac{49}{7} + \frac{5}{7} = \frac{54}{7}$. Since 54 and 7 share no common factors greater than 1, $\frac{m}{n} = \frac{54}{7}$, and therefore, $m + n = 54 + 7 = \boxed{61}$.

39. The probability that Maia goes crazy on the nth day is $(\frac{2}{3})^{n-1} \cdot \frac{1}{3}$, since she has to not go crazy on the first $n-1$ days, and then go crazy the next day. If this happens, she gets $5(n-1)$ potato points. So, the expected number of potato points she gains is

$$\sum_{n=2}^{\infty} \left(\frac{2}{3}\right)^{n-1} \cdot \frac{1}{3} \cdot 5(n-1) = 5 \cdot \frac{1}{3} \cdot \left(\left(\frac{2}{3}\right)^1 \cdot 1 + \left(\frac{2}{3}\right)^2 \cdot 2 + \left(\frac{2}{3}\right)^3 \cdot 3 + \ldots\right)$$

 If we let $S = \left(\frac{2}{3}\right)^1 \cdot 1 + \left(\frac{2}{3}\right)^2 \cdot 2 + \ldots$, then notice that $\frac{2}{3}S = 0 + \left(\frac{2}{3}\right)^2 \cdot 1 + \left(\frac{2}{3}\right)^3 \cdot 2 + \ldots$. So, $S - \frac{2}{3}S = \left(\frac{2}{3}\right)^1 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \ldots$. This is an infinite geometric series, whose sum is $\frac{2/3}{1-\frac{2}{3}} = 2$. So, since $\frac{1}{3}S = 2$, $S = 6$, implying that the original sum is $6 \cdot \frac{5}{3} = 10$, and so the answer is $\boxed{10}$.

40. If we let a_n be the expected monetary gain for Jaiden when the ant is at integer n, we have that $a_n = \frac{1}{2}a_{n-1} + \frac{1}{2}a_{n+1}$ with $a_1 = -17, a_{18} = 17$. However, note that the recurrence relation represents one of an arithmetic sequence, meaning that $a_1, a_2, \ldots$ is an arithmetic sequence. The common difference is $\frac{a_{18} - a_1}{17} = \frac{34}{17} = 2$, so $a_{13} = a_1 + 12 \times 2 = -17 + 24 = \boxed{7}$.