Joe Holbrook Memorial Math Competition

4th Grade

October 11, 2025

1. Note that anything multiplied by 0, so $3 \times 0 = 0$. From there we can compute: $2 - 0 + 4 \cdot 5 = \boxed{22}$.
2. Maddie eats a total of $24 + 35 = \boxed{59}$ muffins.
3. The fruit store has $5 \times 7 = \boxed{35}$ grapes.
4. The bush has 43 - 17= $\boxed{26}$ leaves left.
5. There are 5 vowels every time he says the alphabet, so he says them a total of $5 \cdot 200 = \boxed{1000}$ times.
6. He plays for half an hour twice, so he plays for one hour in total. This is equal to $\boxed{60}$ minutes.
7. A regular pentagon has five equal sides, so a pentagon with side length 3 has a perimeter of $3 \times 5 = \boxed{15}$.
8. The number of pieces I eat is directly proportional to the number of minutes it takes for me to eat them. So, since there are 60 minutes in an hour, and $60 = 5 \times 12$, I can eat $2 \times 12 = \boxed{24}$ pieces in an hour.
9. Converting to the percentages to decimals gives $0.5 \cdot 0.2 \cdot 670 = \boxed{67}$.
10. John can run at a pace of $6\frac{1}{2} = \frac{13}{2}$ minutes per mile, so it will take him $\frac{13}{2} \times 4 = \boxed{26}$ minutes to run 4 miles.
11. The third prime number is five. The third letter of "five" is $\boxed{\text{v}}$.
12. There are 21 people in front of Bob and 8 people behind him. Thus the difference is $21 - 8 = \boxed{13}$.
13. If Michael uses 4 books, he will have 1600 XP, which is not enough. If he uses 5 books, he will have 2000 XP, which is enough. So, he must use $\boxed{5}$ books.
14. To raise 55 by 3 octaves, we need to multiply by two 3 times, or equivalently, multiply by 8. Our answer is $55 \cdot 8 = \boxed{440}$.
15. Since there are 12 inches in a foot, we are scaling the area by a factor of $12^2 = 144$. The area of the rectangle, in square feet, is 40, so the area in square inches is $40 \cdot 144 = \boxed{5760}$.
16. I need to give away at least 10 diamonds. The largest prime number below 10 is 7, so I must give away $20 - 7 = \boxed{13}$ diamonds.
17. Jack eats $\frac{(5^2)\pi}{4} = \frac{25\pi}{4}$ units of pizza, while Jill eats $\frac{(5\sqrt{2})^2\pi}{24} = \frac{25\pi}{12}$ units. The ratio between them is thus $\frac{\frac{25}{4}}{\frac{25}{12}} = \boxed{3}$.
18. Bob walks $5+12 = 17$ feet. Alice walks $\sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$ feet. Bob walks $17-13 = 4$ more feet, giving us a final answer of $\boxed{4}$.
19. Since every cut will add one to the total count, we can just subtract 46 from 84 to get our final answer. Thus, Robert started with $84 - 46 = \boxed{38}$ pieces of wood.
20. Josh does 8 push-ups in 10 seconds, so his rate is $\frac{4}{5}$ push-ups per second. So in 55 seconds, he does $\frac{4}{5} \times 55 = 44$ push-ups. Since he does a clap push-up every 6th push-up, we count how many multiples of 6 are in 44, which is 7. Therefore, he does $\boxed{7}$ clap push-ups.

21. We see that $9^0 \equiv 1 \pmod{10}$, $9^1 \equiv 9 \pmod{10}$, $9^2 \equiv 1 \pmod{10}$, $\ldots$, $9^{2n} \equiv 1 \pmod{10}$, $9^{2n+1} \equiv 9 \pmod{10}$. 512 is 2^9, so any exponent of this would be even. Therefore, the answer is $\boxed{1}$.

22. This is simply asking for the divisors of 2000 greater than or equal to 5. We note that $2000 = 2^4 \times 5^3$, so the number of divisors is $(4+1)\times(3+1) = 20$. The divisors smaller than 5 are 1, 2, and 4, so we subtract these 3. Therefore, our answer is $20 - 3 = \boxed{17}$.

23. Since we are trying to find the hundreds digit of the product, we are just computing

$$2025 \times 2026 \equiv 25 \times 26 \equiv 650 \pmod{1000}.$$

Thus, the hundreds digit is $\boxed{6}$.

24. Let the width of the garden be w meters.

Then the length l is given by:

$$l = 2w + 3$$

The perimeter P of a rectangle is:

$$P = 2(l + w)$$

Given $P = 54$, we can substitute and find:

$$2((2w+3)+w) = 2(3w+3) = 6w + 6 = 54$$

Subtracting 6 from both sides and dividing by 8, we get: $w = \boxed{8}$.

25. In other words, Evan needs $2200 = 220 * 10$ total points across all his games to win his match. Evan has already bowled $150 * 3 = 450$ points, so he needs to bowl $2200 - 450 = 1750$ points in his final 7 games to win, meaning he must average $\frac{1750}{7} = \boxed{250}$ points for his final seven games.

26. Let the side lengths be a and b with $a \geq b$. Then

$$ab = 36, \qquad 2(a+b) = 26 \implies a + b = 13.$$

From $b = 36/a$, substitute into $a + 36/a = 13$:

$$a^2 - 13a + 36 = 0 \implies (a-4)(a-9) = 0 \implies a = 9 \text{ or } 4.$$

Hence the longer side is $\boxed{9}$.

27. Each path consists of 48 moves right and 2 moves up, in some order. The number of distinct paths is the number of ways to choose 2 of the 50 moves to be up, which is:

$$\binom{50}{2} = \frac{50 \times 49}{2} = \boxed{1225}$$

28. Given that each shape has the same perimeter, this perimeter must be divisible by $3, 4, 5,$ and 6. So, the minimum value of this perimeter (which will ultimately yield the minimum side length of the triangle) is $\operatorname{lcm}(3,4,5,6) = 60$. Since we seek the minimum side length of the equilateral triangle, we must divide by 3 to get an answer of $\frac{60}{3} = \boxed{20}$.

29. By the conditions, Adam and Andrew must be seated next to each other, and Adam and Carol must be seated next to each other. Since Adam must have someone adjacent to him on both sides, he must be placed into one of the three middle seats. Now, there are two ways to seat Andrew and Carol next to Adam, and two ways to seat Bobert and David in the remaining seats. Thus there are $3 \cdot 2 \cdot 2 = \boxed{12}$ ways to seat the five.

30. There are $6 \cdot 6 = 36$ possible outcomes for the values on the two die. If at least one the dice reads 6, then the product will be guaranteed to be divisible by 6. There are $6 + 6 - 1 = 11$ ways to do this, subtracting one to ensure that we do not overcount the case where both die read 6. If neither value is 6, then we must have one value be 2 or 4, and the other value be 3. There are $2 \cdot 2 = 4$ to do this. Therefore, there are 15 possible pairs of numbers that will give a product divisible by 6, so our final probability is $\frac{15}{36} = \frac{5}{12}$, and $p + q = 5 + 12 = \boxed{17}$.

31. Let a_n be the number of mushrooms on day n and b_n be the number of spores on day n. Notice that, for $n \geq 2$, since each existing mushroom from day $n-1$ creates a spore on day n: $b_n = a_{n-1}$. Additionally, because each spore from day $n-1$ becomes a mushroom on day n, the total number of mushrooms on day n is just:
$$a_n = a_{n-1} + b_{n-1} = a_{n-1} + a_{n-2}$$
We can recognize that the number of mushrooms on each day forms the Fibonacci sequence! Thus, to find the number of mushrooms on the 10th day, we need the 10th Fibonacci number, which is $\boxed{55}$.

32. We can observe that the Harry Potter series must be treated as one chunk since the books cannot be reordered within the series. Similarly, the math textbooks form another chunk, and the notebooks form a third chunk. Therefore, we have 3 chunks to arrange, which can be done in 3! ways. Within the notebook category, there are 2! ways to arrange the notebooks, and within the textbooks category, there are 3! ways to arrange the textbooks. Thus, the total number of ways is
$$3! \times 2! \times 3! = 6 \times 2 \times 6 = \boxed{72}.$$

33. After 26 repetitions of the alphabet, Aalok will have recited 650 letters, since he only recites 25 characters per repetition. At this point, he has forgotten every letter once, so his next letter will then be the 2nd letter of the alphabet (since he forgets the first!), which is $\boxed{\text{B}}$.

34. Using the point-slope form of a line, we know the two lines have equations $y - 6 = x - 1 \Rightarrow y = x + 5$ and $y - 6 = 2(x - 1) \Rightarrow y = 2x + 4$, respectively. Plugging in $y = 0$ in the equations to both lines, we find $Q(-5, 0)$ and $R(-2, 0)$ to be our x-intercepts. To find the area of $\triangle PQR$, let QR be the base. This base will have length 3, and since point P is 6 units above the x-axis, the height is 6. Thus the area is $\frac{1}{2} \cdot 3 \cdot 6 = \boxed{9}$.

35. Since Jerry says his number is composite, his number must be one of the composite numbers from 1 to 12: 4, 6, 8, 9, 10, 12. All these numbers are divisible by 2 or 3. For Vaishnavi to say their numbers are not relatively prime, no matter which composite Jerry has, her number must be 6 or 12. For Jerry to uniquely determine her number, his number must also be 6 or 12. In either case, their numbers are 6 and 12. So the sum is $6 + 12 = \boxed{18}$.

36. The optimal selection is to take 1 and all primes below 30 for a total of $\boxed{11}$ numbers, since all primes are coprime to each other (and to 1). When a number k is selected, no additional numbers that share prime factors with k can be selected. Therefore it is not worth it to take composite numbers, since they will contain multiple prime factors, leaving us with less options to pick further numbers.

37. For brevity, label the circles $1, 2, 3, 4$, from left to right. Let P be the tangency point between circles 1 and 2, and let Q be the tangency point between circles 2 and 3. Let R be the intersection of AB and circle 2 closer to A. Note AB passes through Q by symmetry, thus PQ is a diameter of circle 2. This implies $\angle QRP = 90°$.

    Define O as the center of circle 1. Since $\angle PQR = \angle OQA$ and $\angle QRP = 90° = \angle QOA$, AA Similarity implies $\triangle QRP \sim \triangle QOA$. Thus,
$$\frac{PR}{QR} = \frac{AO}{QO} = \frac{1}{3}.$$
    Let $PR = x$. Since $PQ = 10 \cdot 2 = 20$, by Pythagorean Theorem we have
$$20^2 = PQ^2 = PR^2 + QR^2 = x^2 + (3x)^2 = 10x^2$$
    so $x = \sqrt{40}$. Now $\ell^2 = QR^2 = (3x)^2 = 9 \cdot 40 = \boxed{360}$.

38. We start with a rainy Monday. So, the chance that Tuesday is rainy is $\frac{7}{10}$ while the chance it turns clear is $\frac{3}{10}$. Now we consider both cases in determining Wednesday's weather. If Tuesday is rainy, then the chance that Wednesday is rainy is $\frac{7}{10}$, while the chance it turns clear is $\frac{3}{10}$. If Tuesday is clear, then the chance that Wednesday is rainy is $\frac{2}{10}$, while the chance it turns clear is $\frac{8}{10}$. So, the probability that Wednesday is rainy is:
$$\frac{7}{10} \cdot \frac{7}{10} + \frac{3}{10} \cdot \frac{2}{10} = \frac{55}{100}.$$

The probability that Wednesday is clear is:

$$\frac{7}{10} \cdot \frac{3}{10} + \frac{3}{10} \cdot \frac{8}{10} = \frac{45}{100}$$

Now we push the calculation one more day. If Wednesday is rainy, it stays rainy on Thursday with probability $\frac{7}{10}$, allowing Thursday to be rainy with a probability of:

$$\frac{55}{100} \cdot \frac{7}{10} = \frac{385}{1000}$$

If Wednesday is clear, it turns rainy with probability $\frac{2}{10}$, allowing Thursday to be rainy with a probability of:

$$\frac{45}{100} \cdot \frac{2}{10} = \frac{90}{1000}$$

So, the probability that Thursday is rainy is $\frac{385}{1000} + \frac{90}{1000} = \frac{475}{1000} = \frac{19}{40}$. Our answer is $19 + 40 = \boxed{59}$.

39.

$$\begin{aligned} \frac{1}{a(b+1)} + \frac{1}{b(a+1)} &= \frac{1}{(a+1)(b+1)}. \\ \Rightarrow \frac{b(a+1)+a(b+1)}{ab(a+1)(b+1)} &= \frac{1}{(a+1)(b+1)}. \\ \Rightarrow \frac{b(a+1)+a(b+1)}{ab} &= 1. \\ \Rightarrow \frac{2ab+a+b}{ab} &= 1 \\ \Rightarrow 2 + \frac{1}{b} + \frac{1}{a} &= 1 \\ \Rightarrow \frac{1}{a} + \frac{1}{b} &= \boxed{-1} \end{aligned}$$

40. We are given that $p(x)$ is a monic quartic polynomial with roots a, b, c, d, and the following conditions:

$$a + b = 1, \quad ab = 2, \quad c + d = 3, \quad cd = 4$$

Since a and b are roots with known sum and product, they satisfy the quadratic:

$$x^2 - (a+b)x + ab = x^2 - x + 2$$

Similarly, c and d are roots of:

$$x^2 - (c+d)x + cd = x^2 - 3x + 4$$

Therefore, the polynomial $p(x)$ with roots $a, b, c,$ and d is the product of these two quadratics:

$$p(x) = (x^2 - x + 2)(x^2 - 3x + 4)$$

To find $p(2)$, we substitute $x = 2$:

$$\begin{aligned} p(2) &= (2^2 - 2 + 2)(2^2 - 3(2) + 4) \\ &= (4)(2) \\ &= \boxed{8} \end{aligned}$$