Joe Holbrook Memorial Math Competition

5th Grade

October 11, 2025

1. Anything multiplied by 0 gives an answer of $\boxed{0}$.
2. There are 5 vowels every time he says the alphabet, so he says them a total of $5 \cdot 200 = \boxed{1000}$ times.
3. David loses 5 pencils and gains 2 for a net loss of 3 pencils. As he starts with 11 pencils, David is left with $11 - 3 = \boxed{8}$ pencils.
4. We have 43 - 17= $\boxed{26}$.
5. Perimeter: $3 \times 5 = \boxed{15}$.
6. 2 pieces every 5 minutes means 4 pieces every 10 minutes. Multiply by 6 to get 24 pieces every 60 minutes, so our answer is $\boxed{24}$.
7. The third prime number is five. The third letter of "five" is $\boxed{\text{v}}$.
8. Converting to the percentages to decimals gives $0.5 \cdot 0.2 \cdot 670 = \boxed{67}$.
9. Since $99\% = 0.99$, the success rate of the pumpkin patch is $1 - 0.99 = 0.01$. So Bob on average suceeds $2025 \cdot 0.01 = 20.25$ times. Thus the answer is $\boxed{20}$.
10. I need to give away at least 10 diamonds. The largest prime number below 10 is 7, so I must give away $20 - 7 = \boxed{13}$ diamonds.
11. 3 large rides will cost 36 dollars, leaving us with 24 dollars for small ones. Kris can afford 3 small rides with this money, so we have a total of $3 + 3 = \boxed{6}$ rides.
12. If Michael uses 4 books, he will have 1600 XP, which is not enough. If he uses 5 books, he will have 2000 XP, which is enough. So, he must use $\boxed{5}$ books.
13. Since there are 12 inches in a foot, we are scaling by a factor of $12^2 = 144$. The area of the rectangle, in square feet, is 40, so the area in square inches is $40 \cdot 144 = \boxed{5760}$.
14. To raise 55 by 3 octaves, we need to multiply by two 3 times, or equivalently, multiply by 8. Our answer is $55 \cdot 8 = \boxed{440}$.
15. On the first night, Tom slept from 10:00 PM to 8:00 AM, which is 10 hours. Since his average sleep over the two nights was 9 hours and 30 minutes, he slept a total of $2 \times 9.5 = 19$ hours. Therefore, on the second night he slept for $19 - 10 = 9$ hours. Adding 9 hours to 11:30 PM, Tom woke up at 8:30 AM on Wednesday. Thus, $a = 8$ and $b = 30$, so $a + b = \boxed{38}$.
16. We have
$$\frac{28+4}{n+1} = 4.$$
Multiplying both sides by $n+1$,
$$28 + 4 = 4(n+1).$$
Simplifying,
$$32 = 4n + 4.$$
Subtracting 4 from both sides,
$$28 = 4n,$$
so
$$n = \frac{28}{4} = \boxed{7}.$$

17. Alice is given $3 \times 2 = 6$ candies. Now, each of her friends gave away 2 candies, so each has $x - 2$ candies (where x is the original amount), and Alice has 6 candies. They all have the same number, so $x - 2 = 6$, which gives $x = 8$. Therefore, each friend originally had $\boxed{8}$ candies.

18. Since every cut will add one to the total count, we can just subtract 46 from 84 to get our final answer. Thus, Robert started with $84 - 46 = \boxed{38}$ pieces of wood.

19. Josh does 8 push-ups in 10 seconds, so his rate is $\frac{4}{5}$ push-ups per second. So in 55 seconds, he does $\frac{4}{5} \times 55 = 44$ push-ups. Since he does a clap push-up every 6th push-up, we count how many multiples of 6 are in 44, which is 7. Therefore, he does $\boxed{7}$ clap push-ups.

20. Note that regardless of the path Alex chooses, he needs to go right 5 times and up 5 times. The order is irrelevant and as long as he goes either right or up each move, his path will be minimal. Thus, the shortest path is $5 + 5 = \boxed{10}$ meters.

21. We seek the remainder when 2025 is divided by 26, since Aalok will sing in cycles of 26 numbers. Since that remainder is 23, the answer should be $\boxed{23}$.

22. The ratio of the hypotenuse to each leg is $\sqrt{2} : 1$, so each leg is $\frac{1}{\sqrt{2}} \times 8 = 4\sqrt{2}$. Thus, the area is $\frac{1}{2}(4\sqrt{2})^2 = \frac{1}{2}(32) = \boxed{16}$ feet squared.

23. One can manually count the number of mushrooms and spores on each day. This gives one 55 mushrooms and 34 spores on day 10. Alternatively, one can recognize that the number of mushrooms on each day forms the Fibonacci sequence. Note we need the 10th Fibonacci number, which is $\boxed{55}$.

24. The volume of a sphere is given by $\frac{4}{3}\pi r^3$, so the balloon explodes when it reaches a volume of 2304π.

    Setting
$$\frac{4}{3}\pi r^3 = 2304\pi,$$
    we can simplify and solve for r:
$$r^3 = 2304 \times \frac{3}{4} = 1728,$$
    so
$$r = \sqrt[3]{1728} = 12 \text{ inches.}$$
    Since the balloon started with a radius of 1 inch and increases by 1 inch every 20 feet, it must increase by 11 inches to reach a radius of 12 inches. Therefore, the balloon explodes at height
$$11 \times 20 = \boxed{220} \text{ feet.}$$

25. Definition: Condition for the chord to pass through the center

    A chord passes through the center of a regular polygon if and only if it is a diameter, i.e., the two vertices are opposite each other.

    Total number of chords
$$\binom{14}{2} = \frac{14 \times 13}{2} = 91$$
    Number of chords passing through the center

    Since the polygon has an even number of vertices, each vertex has exactly one opposite vertex.
$$\text{Number of opposite pairs} = \frac{14}{2} = 7$$
    Probability

$$\text{Probability} = \frac{7}{91} = \frac{1}{13}$$

$1 + 13 = \boxed{14}$

26. The only digits that can work in the ones place are 4, 6, 8, and 9, and 0. We can find the 3-divisible numbers for each case:

$$4 : 124$$
$$6 : 126, 136$$
$$8 : 128, 148, 248$$
$$9 : 139$$

For the 0 case, since any integer except 0 divides 0, we can count the 2-divisible numbers not ending in 0. This is simply $1+1+2+1+3+1+3+2 = 14$. Thus, there are $\boxed{21}$ 3-divisible numbers.

27. This is basically an organized casework check. If we let (L, M, P) be the ordered triple corresponding to how many yellow pigs Al, Amber, and Apollo get respectively, the only ones that work are:

$$(16, 1, 0), (8, 9, 0), (8, 3, 6), (4, 1, 12), (2, 9, 6), (2, 3, 12) \implies \boxed{6}.$$

28. We want to solve $Q(x) = P(P(x)) = 0$. This means that $P(x)$ is a root of $P(x)$, which is 2 or -2. So we solve x:
$P(x) = x^2 - 4 = \pm 2 \implies x = \pm\sqrt{6}$ and $x = \pm\sqrt{2}$. The positive roots are $\sqrt{2}$ and $\sqrt{6}$, so the sum is $\sqrt{2} + \sqrt{6}$, and $m + n = 2 + 6 = \boxed{8}$,

29. After 26 repetitions of the alphabet, Aalok will have recited 650 letters, since he only recites 25 characters per repetition. At this point, he has forgotten every letter once, so his next letter will then be the 2nd letter of the alphabet (since he forgets the first!), which is $\boxed{B}$.

30. The numbers from 2 to 9 are all super divisible (8 numbers). Any two-digit non-primes are super divisible. There are 21 two-digit primes from 10 to 99. Thus, there are

$$(99 - 1 - 21) = 77$$

super-divisible numbers from 1 to 99.

31. The sum of 504's factors is $(1+2+4+8)(1+3+9)(1+7) = 15 \times 13 \times 8 = 1560$. We can subtract the sum of the odd factors of 504 from this sum. $1560 - (1+3+9)(1+7) = 1560 - (13 \times 8) = 1560 - 104 = \boxed{1456}$.

32. A six-digit palindrome is of the form $abccba$, where a, b, and c are digits. From this, we can see that all six-digit palindromes are divisible by 11 by its divisibility rule, so $X = Y$. We can then use constructive counting to find the number of them. There are 9 choices for a (digits 1 through 9), 10 choices for b (digits 0 through 9), and 10 choices for c (digits 0 through 9), so $X = 9 \cdot 10 \cdot 10 = 900$. Thus, our answer is

$$11Y - X = 11X - X = 10X = 10 \times 900 = \boxed{9000}.$$

33. The total area that the cow can access is equal to the area of the large sector of the circle centered at B and the two small sectors of the circles centered at A and C. The large sector has a radius of 4 and an angle of 225°, so its area is $\frac{225}{360}(4)^2(\pi) = 10\pi$. The two small sectors have a radius of 2 and an angle of 45°, so their combined area is $2(\frac{45}{360})(2^2)(\pi) = \pi$. So, the total area that we wanted to find is $10\pi + \pi = 11\pi$, and $k = \boxed{11}$.

34. Expanding both sides of the equation, we have

$$p^2 + p + 1 = q^2 + q + 1 + 242.$$

Bringing all the variable terms to the left-hand side and using difference of squares, we have

$$p^2 - q^2 + p - q = 242 \implies (p-q)(p+q) + (p-q) = 242.$$

This can be factored as

$$(p-q)(p+q+1) = 242.$$

Since both factors on the left-hand side must be integers, it is beneficial to prime factorize 242 as 2×11^2. Setting $p - q = 2$ and $p + q + 1 = 11^2$, and solving this system, we find that $(p, q) = (61, 59)$. Thus, our answer is

$$61 + 59 = \boxed{120}.$$

35. Since Jerry says his number is composite, his number must be one of the composite numbers from 1 to 12: 4, 6, 8, 9, 10, 12. All these numbers are divisible by 2 or 3. For Vaishnavi to say their numbers are not relatively prime, no matter which composite Jerry has, her number must be 6 or 12. For Jerry to uniquely determine her number, his number must also be 6 or 12. In either case, their numbers are 6 and 12. So the sum is $6 + 12 = \boxed{18}$.

36. For brevity label the circles $1, 2, 3, 4$, from left to right. Let P be the tangency point between circles 1 and 2, and let Q be the tangency point between circles 2 and 3. Let R be the intersection of AB and circle 2 closer to A. Note AB passes through Q by symmetry, thus PQ is a diameter of circle 2. This implies $\angle QRP = 90°$.

    Define O as the center of circle 1. Since $\angle PQR = \angle OQA$ and $\angle QRP = 90° = \angle QOA$, AA Similarity implies $\triangle QRP \sim \triangle QAO$. Thus,
$$\frac{PR}{QR} = \frac{OA}{QO} = \frac{1}{3}.$$
    Let $PR = x$. Since $PQ = 10 \cdot 2 = 20$, by Pythagorean Theorem we have
$$20^2 = PQ^2 = PR^2 + QR^2 = x^2 + (3x)^2 = 10x^2$$
    so $x = \sqrt{40}$. Now $\ell^2 = QR^2 = 9 \cdot 40 = \boxed{360}$.

37. At turn one, Chris needs to remove 1 or 10. If he removes 1, then the sum is now 54 and he needs to make it divisible by 8. So, he needs to remove 6. Then, the sum is 48 and he will lose on his next turn. If he removes 10, then the sum is 45 and he needs to make it divisible by 8, so he has to remove 5. The sum is now 40, and he will lose on his next turn. So the total turns at most can be $\boxed{3}$.

38. The shape of these graphs are relatively well-known in competition math. They are squares that are rotated around their centers by 45 degrees, with diagonals of length 6. If the contestant doesn't recognize this, they can also graph each subcase for the absolute values, which will certainly be a little more time consuming. Once they draw out the diagram, it is evident that the second graph is an identical copy of the first, shifted right by 3 units.

    It should be intuitive that their intersection forms a square, but we will prove it in this solution. $\triangle A'D'B' \cong \triangle C'D'B'$ by SSS similarity, so $m\angle A'D'B' = m\angle C'D'B' = 45°$. Using a similar proof, $m\angle ABD = m\angle CBD = 45°$ Since $\overline{AB} \perp \overline{A'D'}$, both $\triangle I_1D'B$ and $\triangle I_2D'B$ are right isosceles, which make $I_1D'I_2B$ a square.

    Since $D'B = 3$, we know that the square has side lengths of $\frac{3\sqrt{2}}{2}$, so the area is $\frac{9}{2}$. Thus, the answer is $\boxed{11}$.

39. We use complimentary counting. Without restrictions, there are $\binom{10}{3} = 120$ different subcommittees. We now break into two cases. In-valid subcommittees can either be composed of 3 consecutive members, or of 2 adjacent members and a third from elsewhere around the table. There are ten 3-consecutive-person subcommittees. There are ten 2-consecutive-person committees, and 6 options for the third member (we must exclude the two already selected members as well as the two adjacent to them on either side), and thus 60 subcommittees with two adjacent members. Thus, our answer is $120 - 10 - 60 = \boxed{50}$ valid subcommittees.

40. Let the midpoint of $\overline{PQ}$ and $\overline{EF}$ be O and G respectively. Note that $\overline{PQ} \parallel \overline{EF}$ so $\triangle AOQ \sim \triangle AGF$. Through this similarity, we have $\frac{AO}{AG} = \frac{OQ}{GF}$. So, $OQ = GF \cdot \frac{AO}{AG}$. Note $GF = \frac{1}{2} \cdot EF = \frac{1}{2} \cdot \sqrt{2}$. Also, $AO = \frac{1}{2}AC = \frac{1}{2} \cdot 4\sqrt{2} = 2\sqrt{2}$ and $AG = AC - CG = 4\sqrt{2} - \frac{1}{2}\sqrt{2} = \frac{7}{2}\sqrt{2}$. Plugging these values in yields $OQ = (\frac{1}{2} \cdot \sqrt{2}) \cdot \frac{2\sqrt{2}}{\frac{7}{2} \cdot \sqrt{2}} = \frac{2\sqrt{2}}{7}$. Thus, $PQ = 2OQ = \frac{4\sqrt{2}}{7}$. Squaring gives $PQ^2 = \frac{32}{49}$ for a final answer of $32 + 49 = \boxed{81}$.