Joe Holbrook Memorial Math Competition

6th Grade

October 10, 2025

1. As $6 + 12 = 18$, they have $\boxed{18}$ donuts.
2. There are 5 vowels every time he says the alphabet, so he says them a total of $5 \cdot 200 = \boxed{1000}$ times.
3. $5 \times 7 = \boxed{35}$ grapes
4. He plays for half an hour twice, so he plays for one hour in total. This is equal to $\boxed{60}$ minutes.
5. Perimeter: $3 \times 5 = \boxed{15}$.
6. The answer is $1 + 2 + 3 + 4 + 5 = \boxed{15}$.
7. 2 pieces every 5 minutes means 4 pieces every 10 minutes. Multiply by 6 to get 24 pieces every 60 minutes, so our answer is $\boxed{24}$.
8. There are 21 people in front of Bob and 8 people behind him. Thus the difference is $21 - 8 = \boxed{13}$.
9. The integers in question are multiples of $2 * 5 = 10$, there are $\boxed{10}$ ranging from 10 to 100.
10. I need to give away at least 10 diamonds. The largest prime number below 10 is 7, so I must give away $20 - 7 = \boxed{13}$ diamonds.
11. 3 large rides will cost 36 dollars, leaving us with 24 dollars for small ones. Kris can afford 3 small rides with this money, so we have a total of $3 + 3 = \boxed{6}$ rides.
12. Since there are 12 inches in a foot, we are scaling by a factor of $12^2 = 144$. The area of the rectangle, in square feet, is 40, so the area in square inches is $40 \cdot 144 = \boxed{5760}$.
13. Alice is given $3 \times 2 = 6$ candies. Now, each of her friends gave away 2 candies, so each has $x - 2$ candies (where x is the original amount), and Alice has 6 candies. They all have the same number, so $x - 2 = 6$, which gives $x = 8$. Therefore, each friend originally had $\boxed{8}$ candies.
14. We see that $9^0 \equiv 1 \pmod{10}$, $9^1 \equiv 9 \pmod{10}$, $9^2 \equiv 1 \pmod{10}$, $\ldots$, $9^{2n} \equiv 1 \pmod{10}$, $9^{2n+1} \equiv 9 \pmod{10}$. 512 is 2^9, so any exponent of this would be even. Therefore, the answer is $\boxed{1}$.
15. This is simply asking for the divisors of 2000 greater than or equal to 5. We note that $2000 = 2^4 \times 5^3$, so the number of divisors is $(4+1) \times (3+1) = 20$. The divisors smaller than 5 are 1, 2, and 4, so we subtract these 3. Therefore, our answer is $20 - 3 = \boxed{17}$.
16. Divisible by 3 is $\left\lfloor \frac{1000}{3} \right\rfloor = 333$. Divisible by 5 is $\left\lfloor \frac{1000}{5} \right\rfloor = 200$. Divisible by 15 is $\left\lfloor \frac{1000}{15} \right\rfloor = 66$. By inclusion-exclusion the ones that are divisible by 3 or 5 is $333 + 200 - 66 = 467$. So, the answer is $1000 - 467 = \boxed{533}$.
17. There are 5 letters in JHMMC, which would give $5! = 120$ arrangements if all were distinct. Since the two Ms are identical, each arrangement is counted twice, so we divide by $2!$. Our final answer is $\frac{120}{2} = \boxed{60}$.
18. Bob walks $5+12 = 17$ feet. Alice walks $\sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$ feet. Bob walks $17 - 13 = 4$ more feet, giving us a final answer of $\boxed{4}$.

19. Let the side lengths be a and b with $a \geq b$. Then

$$ab = 36, \qquad 2(a+b) = 26 \implies a+b = 13.$$

From $b = 36/a$, substitute into $a + 36/a = 13$:

$$a^2 - 13a + 36 = 0 \quad \implies \quad (a-4)(a-9) = 0 \implies a = 9 \text{ or } 4.$$

Hence the longer side is $\boxed{9}$.

20. The ratio of the hypotenuse to each leg is $\sqrt{2} : 1$, so each leg is $\frac{1}{\sqrt{2}} \times 8 = 4\sqrt{2}$. Thus, the area is $\frac{1}{2}(4\sqrt{2})^2 = \frac{1}{2}(32) = \boxed{16}$ feet squared.

21. Each path consists of 48 moves right and 2 moves up, in some order. The number of distinct paths is the number of ways to choose 2 of the 50 moves to be up, that is:

$$\binom{50}{2} = \frac{50 \times 49}{2} = \boxed{1225}$$

22. $lcm(3, 4, 5, 6) = 60 \ \frac{60}{3} = \boxed{20}$

23. One can manually count the number of mushrooms and spores on each day. This gives one 55 mushrooms and 34 spores on day 10. Alternatively, one can recognize that the number of mushrooms on each day forms the Fibonacci sequence. Note we need the 10th Fibonacci number, which is $\boxed{55}$.

24. Suppose my friend takes a random seat. Then, no matter what, there are two seats next to him and three seats not next to him. Since my seating is also random, this gives a probability of $2/5$, which is the same as $\boxed{40}$ percent.

25. Definition: Condition for the chord to pass through the center

A chord passes through the center of a regular polygon if and only if it is a diameter, i.e., the two vertices are opposite each other.

Total number of chords

$$\binom{14}{2} = \frac{14 \times 13}{2} = 91$$

Number of chords passing through the center

Since the polygon has an even number of vertices, each vertex has exactly one opposite vertex.

$$\text{Number of opposite pairs} = \frac{14}{2} = 7$$

Probability

$$\text{Probability} = \frac{7}{91} = \frac{1}{13}, \quad p + q = \boxed{14}$$

26. For a right triangle with legs a, b and hypotenuse c, the inradius is

$$r = \frac{a+b-c}{2}.$$

Here $c = \sqrt{6^2 + 8^2} = 10$, so

$$r = \frac{6+8-10}{2} = \frac{4}{2} = \boxed{2}.$$

27. To find the minimal n such that $n!$ is divisible by $(6!)^2$, we need $n!$ to have at least the prime powers in $6!^2$. $(6!)^2 = 2^8 \times 3^4 \times 5^2$, so $n = \boxed{10}$ is the minimum value of n to be divisible by this.

28. Being the 18th letter of the alphabet, R corresponds to the number 9. Now, considering all increasing geometric sequences containing 9, we realize only the sequence $1, 3, 9$ has all three terms between 1 and 26. The code $1, 3, 9$ corresponds to $\boxed{\text{ZXR}}$.

29. The total area that the cow can access is equal to the area of the large sector of the circle centered at B and the two small sectors of the circles centered at A and C. The large sector has a radius of 4 and an angle of $225°$, so its area is $\frac{225}{360}(4)^2(\pi) = 10\pi$. The two small sectors have a radius of 2 and an angle of $45°$, so their combined area is $2(\frac{45}{360})(2^2)(\pi) = \pi$. So, the total area that we wanted to find is $10\pi + \pi = 11\pi$, and $k = \boxed{11}$.

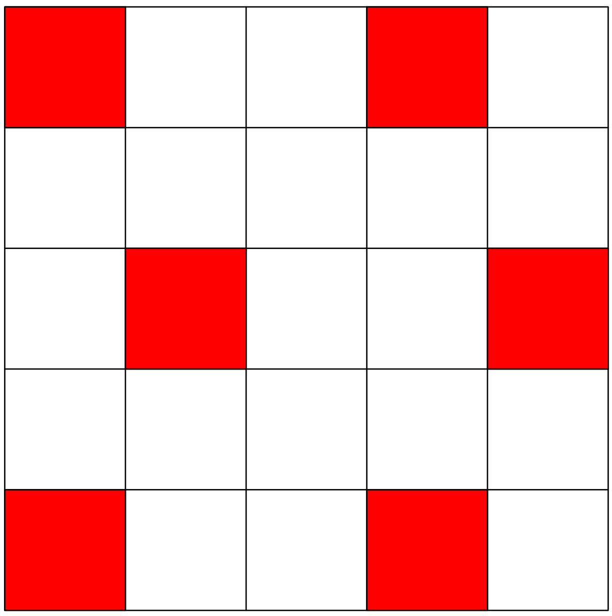
30. Since $\overline{AC}$ is the diameter, it is clear that both triangles $\triangle ABC$ and $\triangle ACD$ are right triangles. From here, by the Pythagorean theorem, we get that $AC = 5$. Using the Pythagorean theorem again, we get $CD = \sqrt{21}$. Thus, $x = \boxed{21}$.

31. We can completely disregard the hour hand and just imagine two minute hands, one moving at the proper speed and one going twice as fast. Under this model, they will intersect once every time the proper speed minute hand revolves around the clock. In 24 hours, this happens $\boxed{24}$ times.

32. Since Jerry says his number is composite, his number must be one of the composite numbers from 1 to 12: 4, 6, 8, 9, 10, 12. All these numbers are divisible by 2 or 3. For Vaishnavi to say their numbers are not relatively prime, no matter which composite Jerry has, her number must be 6 or 12. For Jerry to uniquely determine her number, his number must also be 6 or 12. In either case, their numbers are 6 and 12. So the sum is $6 + 12 = \boxed{18}$.

33. The distance condition means that anything which is two units away or closer is not allowed, but a distance of two squares in one direction and one square in the other direction (a knight's move) is allowed.

    The maximum number is 6, achievable using the following configuration:

    It can be shown that 7 is impossible. One way is to consider all possible configurations of 6 and see that adding another red square is impossible.

34. We are given that $p(x)$ is a monic quartic polynomial with roots a, b, c, d, and the following conditions:

$$a + b = 1, \quad ab = 2, \quad c + d = 3, \quad cd = 4$$

Since a and b are roots with known sum and product, they satisfy the quadratic:

$$x^2 - (a + b)x + ab = x^2 - x + 2$$

Similarly, c and d are roots of:

$$x^2 - (c+d)x + cd = x^2 - 3x + 4$$

Therefore, the polynomial $p(x)$ is the product of these two quadratics:

$$p(x) = (x^2 - x + 2)(x^2 - 3x + 4)$$

We now expand this product:

$$\begin{aligned} p(x) &= (x^2 - x + 2)(x^2 - 3x + 4) \\ &= x^2(x^2 - 3x + 4) - x(x^2 - 3x + 4) + 2(x^2 - 3x + 4) \\ &= x^4 - 3x^3 + 4x^2 - x^3 + 3x^2 - 4x + 2x^2 - 6x + 8 \\ &= x^4 - 4x^3 + 9x^2 - 10x + 8 \end{aligned}$$

To find $p(2)$, we substitute $x = 2$:

$$\begin{aligned} p(2) &= 2^4 - 4(2^3) + 9(2^2) - 10(2) + 8 \\ &= 16 - 32 + 36 - 20 + 8 \\ &= (16 + 36 + 8) - (32 + 20) \\ &= 60 - 52 \\ &= \boxed{8} \end{aligned}$$

35. For brevity label the circles $1, 2, 3, 4$, from left to right. Let P be the tangency point between circles 1 and 2, and let Q be the tangency point between circles 2 and 3. Let R be the intersection of AB and circle 2 closer to A. Note AB passes through Q by symmetry, thus PQ is a diameter of circle 2. This implies $\angle QRP = 90^\circ$.

    Define O as the center of circle 1. Since $\angle PQR = \angle OQA$ and $\angle QRP = 90^\circ = \angle QOA$, AA Similarity implies $\triangle QRP \sim \triangle QAO$. Thus,
    $$\frac{PR}{QR} = \frac{OA}{QO} = \frac{1}{3}.$$
    Let $PR = x$. Since $PQ = 10 \cdot 2 = 20$, by Pythagorean Theorem we have
    $$20^2 = PQ^2 = PR^2 + QR^2 = x^2 + (3x)^2 = 10x^2$$
    so $x = \sqrt{40}$. Now $\ell^2 = QR^2 = 9 \cdot 40 = \boxed{360}$.

36. We have:
    $$\begin{aligned} S &= \frac{1}{2^2 - 1} + \frac{1}{4^2 - 1} + \cdots + \frac{1}{100^2 - 1} \\ &= \frac{1}{(2-1)(2+1)} + \frac{1}{(4-1)(4+1)} + \cdots + \frac{1}{(100-1)(100+1)} \\ &= \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{99 \cdot 101} \\ &= \frac{1}{2}\left(\frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \frac{1}{5} + \cdots + \frac{1}{99} - \frac{1}{101}\right) \\ &= \frac{1}{2}\left(\frac{1}{1} - \frac{1}{101}\right) \\ &= \frac{50}{101} \end{aligned}$$
    Hence the answer is $50 + 101 = \boxed{151}$.

37. The greatest 3-digit number in base 7 is $666_7 = 342_{10}$, so we only need to consider base 10 numbers less than or equal to 342.
    Let the number be of the form $100a + 10b + 5$ in base 10. Since its unit digit in base 7 is 5, we have:
    $$100a + 10b + 5 \equiv 5 \pmod 7 \implies 100a + 10b \equiv 0 \pmod 7.$$

Then we have:

$$2a + 3b \equiv 0 \pmod 7.$$

We now check values of $a \in \{1, 2, 3\}$ and b that satisfy the congruence. Valid combinations of (a, b) are $(1, 4), (2, 1), (2, 8)$. Thus, Noel's favorite numbers are 145, 215, and 285, giving us the final answer of $145 + 215 + 285 = \boxed{645}$.

38. The only result that matters here is the result of the ten-sided die. No matter what values are obtained by the other dice, exactly one result from the ten-sided die will make the total sum congruent to $7 \pmod{10}$. Thus, the probability is $\frac{1}{10}$. Since this fraction is already in lowest terms, we have $m = 1$ and $n = 10$, so $m + n = 11$, giving the answer $\boxed{11}$.

39. If n is the first term of the series and r is the ratio, the sum of the series is $\frac{n}{1-r} = \frac{11 \cdot 13}{4} \to r = 1 - \frac{4n}{11 \cdot 13}$. n now must be divisible by 11 or 13 or r will have a composite number in the denominator. If $n = 11$, $r = 1 - \frac{4}{13} = \frac{9}{13}$ which doesn't work. If $n = 22$, $r = 1 - \frac{8}{13} = \frac{5}{13}$ which works. If $n = 33$, $r = 1 - \frac{12}{13} = \frac{1}{13}$ which doesn't work. If $n = 13$, $r = 1 - \frac{4}{11} = \frac{7}{11}$, which works. If $n = 26$, $r = 1 - \frac{8}{11} = \frac{3}{11}$ which works. Thus the answer is $22 + 13 + 26 = \boxed{61}$.

40. We proceed by first finding $\frac{[ABD]}{[ABC]}$, then finding $\frac{[BED]}{[ABD]}$. Letting the length of the altitude from A to BC be h, we can see that $[ABD] = \frac{(h)(BD)}{2}$, $[ABC] = \frac{(h)(BC)}{2}$, so $\frac{[ABD]}{[ABC]} = \frac{BD}{BC}$. By the Angle Bisector Theorem:

$$\frac{AB}{AC} = \frac{BD}{DC} \to \frac{13}{15} = \frac{BD}{DC}$$

$$\frac{BD}{BC} = \frac{BD}{BD + DC} = \frac{13}{13 + 15} = \frac{13}{28} = \frac{[ABD]}{[ABC]}$$

By similar logic, we can see that $\frac{[BED]}{[ABD]} = \frac{ED}{AD}$. We find this ratio by finding the ratio between the lengths of the altitudes from E to $\overline{BC}$ and from A to $\overline{BC}$, which we previously defined as h (This ratio is equivalent to the one we seek as these altitudes create similar right triangles that allow us to come to this conclusion). Note that since E is the intersection of the angle bisectors of $\angle BAC$ and $\angle ABC$, it is also the incenter of $\triangle ABC$. Thus, the length of the altitude from E to BC is just the inradius of $\triangle ABC$, which we represent as r. The ratio we seek is $\frac{r}{h}$ Both lengths we seek to find can be found through representations of the area of $\triangle ABC$. We see that

$$\frac{14h}{2} = 7h = [ABC] = rs = 21r \to 7h = 21r \to \frac{r}{h} = \frac{1}{3}$$

Thus,

$$\frac{[BED]}{[ABD]} = \frac{ED}{AD} = \frac{r}{h} = \frac{1}{3}$$

Here, we can obtain an answer of

$$\frac{[BED]}{[ABC]} = \frac{[BED]}{[ABD]} \times \frac{[ABD]}{[ABC]} = \frac{1}{3} \times \frac{13}{28} = \frac{13}{84}$$

Giving us a final answer of $13 + 84 = \boxed{97}$