Joe Holbrook Memorial Math Competition

7th Grade

October 12, 2025

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may use the following aids:
  - Pencil or other writing utensil
  - Eraser
  - Blank scrap paper
- You may not use the following aids:
  - The Internet
  - Books or other written sources
  - Other people
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

Other Notes

- All answers are either integers or strings of letters. Make sure you do not make any mistakes when writing your answers, as you will not be given credit if you do so.
- You do not need to write units in your answers.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. Evaluate $2 \times 0 \times 2 \times 5 + 0 \times 2 \times 5 + 2 \times 5 + 5$.

2. Bozhan bought 20 plushies for 3 of his friends. If all of his friends receive the same amount of plushies, what is the least number of plushies Bozhan can keep for himself?

3. David is writing problems for a certain 40-question math contest! If he can write 5 high-quality questions per day, how many days will it take for him to write 40 questions?

4. Matthew, Mark, Luke, and John go on a run together. Of the four, Matthew is the fastest. If they all start running at the same time and place, how many people will be behind Matthew after 5 minutes of running?

5. Emma is in math class and is tasked with evaluating $6-(5+(4-(3+(2-1))))$. When copying down the question from the board, she forgets to write the parentheses. What is the positive difference between Emma's answer and the correct one?

6. Find the greatest sum of the digits of a year after 2000 but before 2025.

7. Alan runs 5 miles every day, with the exception of Saturdays, when he runs 10 miles. How many miles does Alan run per week?

8. Arnav and Gabriele are the guessing the height of a local friendly giant! Arnav guesses the giant has a height of $12'5''$. However, the giant is $13'9''$. If Gabriele wants his guess to be equally as far from the giant's height as Arnav's, what height should he guess in inches?

9. Bob runs twice as fast as Alex, who runs six times slower than Joe. If Joe and Bob each run a marathon and Joe finishes in 210 minutes, how many minutes does it take Bob?

10. What is the greatest common factor of 625 and 2025?

11. In Minecraft, a day is 20 real-life minutes. How many Minecraft days fit in a real-life day? (Assume that a real-life day is 24 hours.)

12. Olivia lives 9.0 kilometers from BCA and Esther lives 6.5 kilometers from BCA. What is the positive difference between the maximum and minimum possible distances between them in kilometers?

13. The clock strikes 7 o'clock in Tableland. After 2025 hours, what number is pointed to by the hour hand?

14. A regular heptadecagon (17-sided polygon) is spun around its center, and the distance from its center to any vertex is 42. If the area of the region swept out by the moving polygon is $a\pi$, find a.

15. Bob's painting is placed in a frame that forms a border that is 1 inch thick on all sides. Given that the painting itself has a length twice as long as its width and has a perimeter of 54 inches, find the area of the frame in square inches.

16. How many digits does $2^{22} \cdot 5^{10}$ have?

17. Jose rolls two six-sided dice. Let the probability that one of the numbers he rolls is a factor of the other number be p. Find $36p$.

18. Rumi needs help finding her hotel room! Every room has a unique 4-digit number and Rumi has some clues as to what her room number is. She knows that:

    - The digit in the thousands place is 4.
    - The room number is odd.
    - The digit in the tens place is a factor of 24.
    - Her friends Mira and Zoey are staying in rooms 4028 and 4029 (and thus, Rumi cannot be in either of those rooms).

    Based on these clues, how many possible rooms does Rumi have to try (including her own room)?

19. Points A, B, C, D, and E lie on a line in that order. The lengths AB, BC, CD, DE form an arithmetic sequence in this order. If $AC = 11$ and $BD = 9$, find the length of $\overline{BE}$.

20. A number is called n-divisible if it has n digits and each subsequent digit is divisible by the previous digit. Additionally, no digits can be repeated. For example, 124 is 3-divisible, but 356 is not. How many 3-divisible numbers are there?

21. Aalok is playing tennis! The probability of success in making his first serve is 60%. If he misses his first serve, he will then attempt a second serve, which he will make 90% of the time. The probability that Aalok does not miss both serves can be represented as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

22. Call a positive integer *nearly prime* if it has exactly three distinct positive integer divisors. Find the sum of all nearly prime integers less than 50.

23. A 10 foot ladder leans against a wall such that the base of the ladder rests 8 feet away from the base of the wall. I want to adjust the ladder so that I can climb up to the top of the ladder and be exactly 5 feet above the floor. If the absolute distance I have to move the base of this ladder to achieve this can be expressed as $b\sqrt{c}-a$ where c has no perfect square factors greater than 1, find $a+b+c$.

24. Joey starts at $(0,0)$ and wants to reach BCA at $(3,4)$. He can only move up or to the right and each move goes from one lattice point to an adjacent lattice point. He must change his direction exactly 2 times along the way. How many different paths can he take to reach BCA?

25. Bob used 6 fence posts, each connected by 1 foot long segments, to form a closed figure. The maximum area Bob could enclose can be written as $\frac{b\sqrt{c}}{a}$, where c has no perfect square factors greater than 1 and $\gcd(a,b)=1$. What is $a+b+c$?

26. Let V be the volume of the largest cube that can fit in a sphere of radius 3. Compute V^2.

27. How many positive integers less than 1000 are divisible by exactly one of 2 or 5?

28. In Dr. Abramson's logic class, everyone is either a truth teller or a liar. They each make the following statements:

    (a) Arnav: "If Daniel is a truth teller, Gabriele is a liar."
    (b) Bryan: "There are exactly 3 people lying."
    (c) Chris: "Arnav and I are the same kind of person."
    (d) Daniel: "Both Chris and Gabriele are truth tellers."
    (e) Fiona: "At least one of Bryan or Gabriele is a liar."
    (f) Gabriele: "Daniel is a liar."

    List the first letters of the names of the truth tellers in alphabetical order. For example, if everyone was telling the truth, the answer would be ABCDFG.

29. I have 11 balls, each with a different number from the set $\{1,2,4,8,\ldots,1024\}$. I then sort these balls into two different boxes and find the sums of the numbers in each box, called A and B. How many different values can $A-B$ take?

30. A quadratic with a leading coefficient of 7 has a remainder of 8 when divided by either $(x-2)$ or $(x+2)$. Evaluate $f(1)$.

31. A group of 3 explorers are cave diving! They face 10 tunnels directly connected to their current position, exactly one of which contains the exit at random. Exploring a tunnel and returning to report together takes 1 minute, regardless of how many people enter. When someone finds the exit, they return to the main area to report. If they remember which tunnels have been explored and coordinate optimally, what is the expected time in seconds until the group knows where the exit is?

32. An analog clock starts at $12:00$ (both hands starting at the 12 o'clock position). To the nearest integer, how many minutes past the hour will the time of the 7th overlap be? Note that the starting position does not count as an overlap.

33. Let the infinite sum of the reciprocals of all positive integers that are products of powers of 2 and 3 be

$$S = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{6}+\cdots.$$

    Evaluate S.

34. A factor of $10!$ is chosen at random. The probability that it is a perfect square can be written as $\frac{m}{n}$, where $\gcd(m,n)=1$ and m and n are nonnegative integers. Find $m+n$.

35. There are 20 people at a party. Every person chooses one other person (no one may choose themselves) uniformly at random and gives that person a gift. Two people form a *compatible pair* if they choose each other (i.e., they exchange gifts). The expected number of compatible pairs can be written as $\frac{m}{n}$, where $\gcd(m, n) = 1$ and m and n are nonnegative integers. Find $m + n$.

36. Lam rolls three fair six-sided dice and announces their sum to Colin. The probability that Colin can determine at least one of the dice rolls with certainty can be written as $\frac{m}{n}$, where $\gcd(m, n) = 1$ and m and n are nonnegative integers. Find $m + n$.

37. A sphere with radius 1 is inscribed in a square pyramid with a base of side length 4. The pyramid's volume can be written as $\frac{m}{n}$, where $\gcd(m, n) = 1$ and m and n are nonnegative integers. Find $m + n$.

38. My math teacher has 24 identical copies of an algebra textbook and 21 identical copies of a geometry textbook. They are arranged in a line on a shelf. Let N be the number of unique arrangements. What is the number of consecutive zeroes at the end of the decimal representation of N?

39. Let $P(x)$ be a polynomial with degree 6 and leading coefficient 1 with $P(0) = 10$, $P(1) = 8$, $P(2) = 6$, $P(3) = 4$, $P(4) = 2$, and $P(5) = 0$. Find $P(6)$.

40. How many ways are there to color the cells of a 5×5 grid black and white such that every 2×2 subgrid contains an odd number of black cells?