Joe Holbrook Memorial Math Competition

8th Grade

October 12, 2025

1. Mark, Luke, and John are behind Matthew so there are $\boxed{3}$ people.
2. We just need to compute the remainder when 20 is divided by 3, which is 2. Since the remainder is less than the divisor, we can be sure that we cannot give more plushies. Thus, our answer is $\boxed{2}$.
3. Each cut simply splits one layer into two, so each cut increases the total number of pieces by 1. So, $5 + 1 = \boxed{6}$
4. $60 \div 12 = \boxed{5}$.
5. Alex types the word ""notebook"", which has 8 characters, in 3 seconds. So Alex's typing rate is 160 characters per minute. Thus, $\boxed{\text{Alex}}$ types faster.
6. We have
$$\frac{2^4}{3^5} \times \frac{3^6}{2^2} = 2^{4-2} \times 3^{6-5} = 2^2 \times 3 = \boxed{12}.$$
7. In the 220 seconds it takes for Steve to finish his lap, Raul can run $\frac{220}{20} = \boxed{11}$ laps.
8. By using 3 dimes and a quarter $(3 * 10 + 25 = 55)$, I can use a minimum of $\boxed{4}$ coins.
9. The smallest prime larger than 6, 7 is 11. The least common multiple of 6, 7, and 11 is 462. The multiple of 462 greater than 500 less than 1000 is $462 \times 2 = \boxed{924}$.
10. She can wear 3 red outfits, 4 blue outfits, and $2 \cdot 5 = 10$ green outfits. We have $3+4+10 = \boxed{17}$ total outfits.
11. Note that the units digits of all the addends greater than or equal to 5! is 0, since all of them will contain at least one factor of 5 and 2. This makes them multiples of 10, which always end in a 0. Since a 0 will not contribute to our units digit sum, our answer is just the units digit of $1! + 2! + 3! + 4!$, which is $\boxed{3}$.
12. The region that is swept is a circle, and the distance from the center to the vertex is the circle's radius. Thus, the area is $\pi r^2 = \pi(42)^2 = 1764\pi$, so our answer is $\boxed{1764}$.
13. The formula $\dfrac{180(n-2)}{n}$ gives the measure of an interior angle of a regular polygon, and we can see that at $n = \boxed{10}$ the interior angle is $144°$.
14. Note that the sum remains unchanged every minute because the value of the removed integers is the same as the value of the added sum. After 5 minutes, the sum stays at $\dfrac{10 \cdot 11}{2} = 55$. Additionally, every minute, two integers are removed and only one is added. After 5 minutes, five integers are removed and five are left. Thus, our answer is $55 \cdot 5 = \boxed{275}$.
15. Note that $343 = 7^3$. The sum of the positive integer divisors is the sum of the positive powers of 7 up to 343. Namely, $1 + 7 + 49 + 343 = \boxed{400}$.
16. All $\boxed{4}$ rotations can be done. In fact, all four rotations can be achieved in four different ways. A possible way to achieve them is: 0 degrees: reflection over x followed by x. 90 degrees: reflection over x followed by reflection over y=x. 180 degrees: reflection over x followed by reflection over y. 270 degrees: reflection over x followed by reflection over y = -x.
17. We simply need to count how many of the 36 pairs of rolls he can achieve satisfy our condition. For example, we would first count $(1,1), (1,2), \ldots (1,6)$, then count $(2,1), (2,2), (2,4), (2,6)$, and so on. This gives us a probability of $\dfrac{22}{36}$ for a final answer of $\boxed{22}$.

18. We think of the total number of cases to be those where we check numbers of only 1 to 4. This way, we know all the cases will always have a maximum ball number of 4 or less. There are $4\times4\times4\times4\times4 = 1024$ ways to do this. We will subtract the number of cases where Danny did not check a 4 which is $3\times3\times3\times3\times3 = 243$. Hence, our answer is $1024 - 243 = \boxed{781}$.

19. Let $BC = x$, then $AB = x+k$ and $CD = x-k$ for some k, since the lengths form an arithmetic sequence. We are given that $AC = AB + BC = 2x + k = 11$ and $BD = BC + CD = 2x - k = 9$. Solving these equations gives $x = 5$ and $k = 1$. Thus, $BE = BC + CD + DE = x + (x - k) + (x - 2k) = 3x - 3k = 15 - 3 = \boxed{12}$.

20. It can be shown that $s_{2n+1} = 2^{n-1}$. Therefore,
$$s_{19} = s_{2\cdot9+1} = 2^8 = \boxed{256}.$$

21. By definition of the tangent, we may drop a perpendicular from O to the point of tangency. So, the chord forms a right angle with the perpendicular. The length of the perpendicular equals the smaller circle's radius, 5. Without loss of generality, let us call the segment connecting the point of tangency with one end of the chord as x. Notice that x and the perpendicular form a right triangle, with the hypotenuse being the radius of the larger circle! So, the hypotenuse has length 13. Using the Pythagorean theorem, we find that $x = 12$. We'll find that repeating the same process for the other segment, which connects the point of tangency with the other end of the chord, gets us the same result: $x = 12$. Therefore, the length of the chord is $12 + 12 = \boxed{24.}$

22. For a base b, in order for a number to have exactly 3 digits, it should be in the range $[b^2, b^3)$. Using these restrictions, we find that the number should be in the range $[49, 343)$, $[64, 512)$, $[81, 729)$. The intersection of these sets is $[81, 342]$. Thus, there are $342 - 81 + 1 = \boxed{262}$ such numbers.

23. The closed form for S_n is $S_n = \dfrac{n(n+1)}{2}$. Suppose we have $\dfrac{n(n+1)}{2} = c^2$. Then, we are trying to solve for integer n such that $n^2+n-2c^2 = 0$. The discriminant is $1+8c^2$ and must be a perfect square. Looking at squares that are 1 modulo 8 gives the smallest non-one solution to be $c = 6$. Plugging it in, we get $n = \boxed{8}$.

24. We compute some small values of 18^k to find a pattern:
$$\begin{aligned} 18^1 &\equiv 8 \pmod{10} \\ 18^2 &\equiv 4 \pmod{10} \\ 18^3 &\equiv 2 \pmod{10} \\ 18^4 &\equiv 6 \pmod{10} \\ 18^5 &\equiv 8 \pmod{10} \\ 18^6 &\equiv 4 \pmod{10} \end{aligned}$$
We see that the units digit repeats in cycles of 4: the remainder when 17^{16} is divided by 4 will thus determine the units digit. Notice that $17 \equiv 1 \pmod 4$, so $17^{16} \equiv 1^{16} \equiv 1 \pmod 4$. That means that the units digit of $18^{17^{16}}$ is the same as the units digit of 18^1, which is $\boxed{8}$.

25. Let the common value of both expressions be x. Then from the first nested radical, we have
$$x = \sqrt{A + x} \implies x^2 = A + x \implies A = x^2 - x.$$
From the second nested radical, we have
$$x = \sqrt{B - x} \implies x^2 = B - x \implies B = x^2 + x.$$
Given $A + B = 72$, substituting the expressions for A and B,
$$(x^2 - x) + (x^2 + x) = 2x^2 = 72 \implies x^2 = 36 \implies x = 6 \quad (x > 0).$$
Finally, substituting back to find B,
$$B = x^2 + x = 36 + 6 = \boxed{42}.$$

26. We will try to split 7 move slots into three sections. The possible cases are (1, 1, 5), (1, 2, 4), (1, 3, 3), (2, 2, 3). However, (1, 3, 3) does not have a unique maximum move used so we will only look at (1, 1, 5), (1, 2, 4), and (2, 2, 3). The largest number for each triplet will be attributed to Purple. Only for (1, 2, 4) do we need to consider the order of Red and Blue which is 2 cases. We find the number of combinations to be $\binom{7}{1} \cdot \binom{6}{1} \cdot \binom{5}{5} + 2 \cdot \binom{7}{1} \cdot \binom{6}{2} \cdot \binom{4}{4} + \binom{7}{2} \cdot \binom{5}{2} \cdot \binom{3}{3} = 42 + 210 + 210 = \boxed{462}$.

27. This is a Simon's Favorite Factoring Trick problem, so we can rewrite it as $(x-8)(y+7) = 153 = 3^2 \times 17$. Since there are 6 factors that divide 153, we would think that there would be 6 solution pairs. However, since both x and y are positive, $y + 7$ cannot be equal to 1 or 3. Thus, there are $\boxed{4}$ solution pairs.

28. Place the ground plane P at $z = 0$. Because each sphere has radius 1, its center is 1 unit above P, so the three centers lie in the plane $z = 1$. Pairwise tangency forces the centers to form an equilateral triangle of side 2.

    Let the centroid of this triangle be $T = (0, 0, 1)$ and choose one center as $C = \left(0, -\frac{2}{\sqrt{3}}, 1\right)$. The plane Q parallel to P and tangent to the spheres is 1 unit above every center, hence Q is given by the equation $z = 2$. It touches the sphere at its "north pole" $A = \left(0, -\frac{2}{\sqrt{3}}, 2\right)$.

    Compute

$$D = AT = \sqrt{(0-0)^2 + \left(-\tfrac{2}{\sqrt{3}} - 0\right)^2 + (2-1)^2} = \sqrt{\tfrac{4}{3} + 1} = \frac{\sqrt{21}}{3}.$$

    The answer is $3D^2 = \boxed{7}$.

29. Note that ABC can be split into two $3-4-5$ triangles connected along the side of length 4. One formula that relates the circumradius to known values in the triangle is $[ABC] = \frac{abc}{4R}$ where $[ABC]$ is the area of ABC, a, b, c are the side lengths of the triangle and R is the circumradius. Note that $[ABC] = 2 \cdot \frac{3 \cdot 4}{2} = 12$. So, $R = \frac{abc}{4[ABC]} = \frac{(5)(5)(6)}{(4)(12)} = \frac{25}{8}$. Another way to solve for the circumradius would be to note the circumcenter lies on the altitude from A to BC, break the altitude into R and $4 - R$, and use the Pythagorean Theorem to determine R. With the value of the circumradius, note that $AD = 2R = \frac{25}{4}$. Additionally, note $\angle ABD = 90^\circ$ as it subtends the diameter. So, $BD = \sqrt{AD^2 - AB^2} = \sqrt{\frac{625}{16} - \frac{400}{16}} = \frac{15}{4}$. The final answer is $15 + 4 = \boxed{19}$.

30. We count the number of permutations with HAM, then subtract the number of permutations with both HAM and $MEAT$.

    There are $8-3+1 = 6$ places to insert HAM into the permutation, and $\frac{5!}{2!}$ ways to permute the remaining letters, which yields $6 \cdot \frac{5!}{2} = 360$.

    Now if both HAM and $MEAT$ form a contiguous string $HAMEAT$, we have $8-6+1 = 3$ ways to insert this into the sequence, and 2! ways to permute the remaining letters. However, the strings can also be disjoint. There are 2! ways to determine which order HAM and $MEAT$ go, and 3 ways to insert these strings into the sequence. The last letter is fixed. So in total, we have $3 \cdot 2! + 3 \cdot 2! = 12$.

    Our final answer is $360 - 12 = \boxed{348}$ permutations.

    (verified by python program as well)

31. *Solution 1:* After some simplification, the expression we are tasked to find can be represented in the form:

$$2025\left(\frac{rs + qs + qr + ps + pr + pq}{pqrs}\right)$$

    Applying Vieta's formulas, we obtain an answer of:

$$2025\left(\frac{187}{2025}\right)$$

    giving us a final answer of $\boxed{187}$.

Solution 2: After factoring out the 2025 in the expression, we obtain:

$$2025\left(\frac{1}{pq}+\frac{1}{pr}+\frac{1}{ps}+\frac{1}{qr}+\frac{1}{qs}+\frac{1}{rs}\right)$$

This is just the second symmetric sum of the terms $\{\frac{1}{p},\frac{1}{q},\frac{1}{r},\frac{1}{s}\}$. We can find a polynomial with those four terms as its roots be reversing the order of the coefficients of the presented quadratic. So, the polynomial with the roots $\frac{1}{p},\frac{1}{q},\frac{1}{r},\frac{1}{s}$ is just: $2025x^4+108x^3+187x^2-67x+1$. Now we can simply apply Vieta's formulas, to obtain an answer of $2025(\frac{187}{2025})=\boxed{187}$.

32. The minimum value is 17, achieved by $(c,h,j,m)=(4,3,2,4)$.

To see why, note that the equation can be written as

$$j\times h\times m^2=100-c.$$

Since c,h,j,m are positive integers, c must be at most 6 because for larger c values, the product jhm^2 becomes too small to allow for smaller sums.

By checking possible values of $c=0,1,\ldots,6$ and factoring $100-c$ accordingly, we seek to minimize

$$j+h+2m+c.$$

The minimal value is found at $c=4$, $m=4$, $j=2$, and $h=3$, giving

$$2+3+2\times4+4=\boxed{17}.$$

This problem requires examining factorizations and balancing variables to minimize the sum, rather than assuming values like $7\times7\times2\times1+2$ are optimal.

33. If Daniel is telling the truth, then Gabriele is telling the truth, which means Daniel is lying. That means Gabriele is telling the truth and Daniel is lying, so Chris must also be lying. Then Arnav must be telling the truth. If Fiona is telling the truth, then Bryan must be lying, but then exactly 3 people would be lying, so Bryan would be telling the truth. Therefore Fiona must be lying, and the answer is $\boxed{ABG}$.

34.

$$\begin{aligned}a_n&=a_{n-1}+2a_{n-2}+\cdots+na_0\\&=2a_{n-1}+a_{n-2}+a_{n-3}+\cdots+a_0\\&=3a_{n-1}-a_{n-2},\\F_n&=F_{n-1}+F_{n-2}\\&=2F_{n-2}+F_{n-3}\\&=3F_{n-2}-F_{n-4}.\end{aligned}$$

It can be seen that $a_n=F_{2n-1}$ for all $n>0$. Therefore,

$$a_{2025}=F_{4049}.$$

35. Instead of trying to count this directly, let's first give 8 donuts to all the children. Then we have given $8(4)-26=6$ donuts too many, so we need to count the number of ways we can take away 6 donuts from 4 children. This is now a standard stars-and-bars setup, so our answer is $\binom{6+4-1}{4-1}=\binom{9}{3}=\boxed{84}$.

36. If every number comes up mod 999, 1000, and 1001, d must be relatively prime with each number. $999=3^3\cdot37$, $1000=2^3\cdot5^3$, $1001=7\cdot11\cdot13$. Therefore the smallest possible value of d is $\boxed{17}$.

37. Let point G be the intersection of $\overline{AC}$ and $\overline{BD}$. Since $\overline{BD}$ is the angle bisector of $\angle ABC$, by the Angle Bisector Theorem,

$$\frac{AG}{GC}=\frac{AB}{BC}=\frac{5}{6}.$$

We are given that $\overline{AD}\parallel\overline{BC}$, so by AA similarity, $\triangle DAG\sim\triangle BCG$. Then,

$$\frac{AG}{GC}=\frac{AD}{BC}\implies\frac{5}{6}=\frac{AD}{12}\implies AD=10.$$

Thus, $\triangle ABD$ is isosceles with $AB = AD = 10$. Because $\triangle ABD$ is isosceles and $\overline{AD} \parallel \overline{BC}$, the angles satisfy:

$$\angle ABD = \angle ADB = \angle CBD = \angle EDB.$$

Hence, segments $BE = DE = 10$. Since $BC = 12$,

$$CE = BC - BE = 12 - 10 = 2.$$

Now consider triangles $\triangle ADF$ and $\triangle CEF$. Since $\angle ADF = \angle CEF$ and $\angle DAF = \angle ECF$, we have $\triangle ADF \sim \triangle CEF$ by AA similarity. Therefore,

$$\frac{CF}{AF} = \frac{CE}{AD} = \frac{2}{10} = \frac{1}{5}.$$

Thus, we get a final answer of $1 + 5 = \boxed{6}$.

38. We first consider the probability that a single booth is occupied at $10:30$ AM. For the single booth to be occupied at that time, the group occupying the booth at $10:30$ enters the booth at either $9:00$ AM or $10:00$ AM. For the booth to be first occupied at $9:00$ AM, the unoccupied booth at $8:00$ AM must first remain unoccupied, giving us a total probability of $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ for this case. In the case where the booth becomes occupied at $10:00$ AM, two subcases exist: one where it is empty until $10:00$ AM with a $\frac{1}{8}$ probability and one where it is first occupied from $8:00$ to $10:00$, then re-occupied at $10:00$, which has a $\frac{1}{4}$ probability. Thus, a single booth has a $\frac{1}{4} + \frac{1}{4} + \frac{1}{8} = \frac{5}{8}$ chance of being occupied at $10:30$ AM. By the Principle of Inclusion-Exclusion (PIE), the probability that at least one booth is open at $10:30$ has a value of $1 - \left(\frac{5}{8}\right)^3 = \frac{899}{1024}$, netting us an answer of $\boxed{899}$.

39. Notice that $\angle ACE \cong \angle ABC$ by the Inscribed Angle Theorem. Since right angles $\angle ADB$ and $\angle AEC$ are congruent, by AA similarity, $\triangle AEC \sim ADB$. Also notice that by Heron's formula: $[ABC] = \sqrt{21 * 8 * 7 * 6} = 84$, from which we can find the length AD:

$$\frac{BC \times AD}{2} = 84 \rightarrow AD = 12$$

This also tells us that $BD = 5$, as right triangle $\triangle ADB$ is a $5 - 12 - 13$ right triangle. Thus:

$$CE = \frac{5}{13} \times 15 = \frac{75}{13}, AE = \frac{12}{13} \times 15 = \frac{180}{13}$$

It can also be seen that since $\angle ADC + \angle AEC = 90^\circ + 90^\circ = 180^\circ$, quadrilateral $AECD$ is cyclic, and we can apply Ptolemy's theorem to obtain:

$$AC \times DE = AE \times DC + AD \times CE \rightarrow 15(DE) = \frac{180 * 9}{13} + \frac{75 * 12}{13} \rightarrow DE = \frac{168}{13}$$

giving us a final answer of $168 + 13 = \boxed{181}$

40. We observe that these equations correspond to coefficients in the expansion of the product of two quadratic polynomials:

$$(x^2 + ax + c)(x^2 + bx + d) = 0.$$

Expanding, we get:

$$x^4 + (a + b)x^3 + (ab + c + d)x^2 + (ad + bc)x + cd = 0.$$

Using the given system, this becomes:

$$x^4 + 10x^3 + 33x^2 + 44x + 20 = 0.$$

Next, we factor this quartic polynomial. The factorization is:

$$(x + 1)(x + 2)(x + 2)(x + 5) = 0.$$

Grouping the factors into two quadratics, possible pairs are:

$(x+1)(x+2) = x^2+3x+2, (x+2)(x+5) = x^2+7x+10and(x+1)(x+5) = x^2+6x+5, (x+2)(x+2) = x^2+4x+4$

Matching these with $x^2 + ax + c$ and $x^2 + bx + d$, we have two sets:

1) $(a, c) = (6, 5)$ and $(b, d) = (4, 4)$, sum $a + b + c + d = 19$.

2) $(a, c) = (3, 2)$ and $(b, d) = (7, 10)$, sum $= 22$.

Therefore, the smallest value of $a + b + c + d$ satisfying the system is $\boxed{19}$.