Joe Holbrook Memorial Math Competition

8th Grade

October 12, 2025

General Rules

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.
- Only answers recorded on the answer sheet will be graded.
- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.
- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.
- You may use the following aids:
  - Pencil or other writing utensil
  - Eraser
  - Blank scrap paper
- You may not use the following aids:
  - The Internet
  - Books or other written sources
  - Other people
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

Other Notes

- All answers are either integers or strings of letters. Make sure you do not make any mistakes when writing your answers, as you will not be given credit if you do so.
- You do not need to write units in your answers.
- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. Matthew, Mark, Luke, and John go on a run together. Of the four, Matthew is the fastest. If they all start running at the same time and place, how many people will be behind Matthew after 5 minutes of running?

2. Bozhan bought 20 plushies for 3 of his friends. If all of his friends receive the same amount of plushies, what is the least number of plushies Bozhan can keep for himself?

3. If you make 5 straight cuts across a rectangular chocolate bar, each cut going completely through and not intersecting any other cut, how many pieces will you have?

4. There are 60 donuts at the BCA math team meeting and they are divided evenly among the 12 math team captains. How many donuts does each captain get?

5. Jake claims he can type 150 characters per minute, while Alex claims he can type the word "notebook" once every 3 seconds. Who types faster?

6. Compute $\frac{2^4}{3^5} \times \frac{3^6}{2^2}$.

7. Raul is a very fast runner and thus, can run a lap around a track in just 20 seconds. Steve, being a slower runner, take 220 seconds to run around the same track. Given the two start running at the same time from the same place, how many laps does Raul finish by the time Steve finishes one lap?

8. If I want to make 55 cents and I only have pennies, dimes, and quarters, what is the minimum number of coins I can use?

9. Donald and Ivana go on a walk together. Donald tells Ivana to guess a number he is thinking. Ivana says "6? 7?" and to that Donald replies, "My number is divisible by both those numbers and the smallest prime larger than those two numbers." If Donald's number is larger than 500 and less than 1000, what is it?

10. Alice has four distinguishable hats. A red one, a blue one and 2 green ones. She also has 3 red shirts, 4 blue shirts, and 5 green shirts. If her hat always matches her shirt, and she always wears a shirt and a hat, how many outfits can she wear?

11. What is the units digit of $1! + 2! + 3! + 4! + \cdots + 2025!$?

12. A regular heptadecagon (17-sided polygon) is spun around its center, and the distance from its center to any vertex is 42. If the area of the region swept out by the moving polygon is $a\pi$, find a.

13. What is the smallest value of n such that a regular n-gon has an interior angle greater than $142°$?

14. Judy has the positive integers up to 10 written on a chalkboard. Every minute, she takes two random numbers on the board and replaces them with their sum. After 5 minutes, if A is the sum of the integers on the board and B is the number of integers remaining on the board, find AB.

15. Find the sum of the positive integer divisors of 343.

16. A square centered at the origin with sides parallel to the axes can be rotated about the origin by 0, 90, 180, or 270 degrees counterclockwise about the origin while preserving its shape. It can also be reflected along the x axis, y axis, the line $y = x$, or the line $y = -x$ while preserving its shape. How many of the four aforementioned rotations can be made by performing two (not necessarily distinct) reflections, one after another?

17. Jose rolls two six-sided dice. Let the probability that one of the numbers he rolls is a factor of the other number be p. Find $36p$.

18. In a box there are balls labeled from 1 to 8. Danny takes out a ball, checks it, and then returns it to the box. If he does this to check 5 balls, how many ways are there such that the largest number he checked was 4 if the order in which he picks the balls matters?

19. Points A, B, C, D, and E lie on a line in that order. The lengths AB, BC, CD, DE form an arithmetic sequence in this order. If $AC = 11$ and $BD = 9$, find the length of $\overline{BE}$.

20. A sequence is defined as follows:
$$s_1 = 1, \quad s_2 = 1, \quad s_3 = 1.$$
For $i \geq 4$,
$$s_i = s_1 + s_2 + \cdots + s_{i-2} - s_{i-1}.$$
For example,
$$s_4 = s_1 + s_2 - s_3 = 1,$$
$$s_5 = s_1 + s_2 + s_3 - s_4 = 2.$$
What is s_{19}?

21. Two concentric circles share the same center O. The smaller circle has a radius of 5, while the larger circle has a radius of 13. A chord of the larger circle is drawn so that it lies tangent to the smaller circle. Determine the length of the chord.

22. How many positive integers have exactly 3 digits in their base 7, 8, and 9 representations?

23. Let S_n be the sum of all the positive integers up to n inclusive. Find the smallest $n > 1$ such that S_n is a perfect square.

24. What is the units digit of $18^{17^{16}}$?

25. The value of the infinite nested radical
$$\sqrt{A + \sqrt{A + \sqrt{A + \dots}}}$$
is equal to the value of the infinite nested radical
$$\sqrt{B - \sqrt{B - \sqrt{B - \dots}}}.$$
If $A + B = 72$, what is the value of B?

26. Lemon is trying to create a sequence of 7 colors, but each slot can be one of three colors: Red, Blue, or Purple. Each color must be used at least once, and Purple must be used the most out of the three colors. How many ways can she do this?

27. Find the number of solution pairs (x, y) where x and y are two positive integers such that $xy + 7x - 8y = 209$.

28. Three spheres of radius 1 are all tangent to the same plane P and tangent to each other. A second plane $Q \neq P$ is parallel to plane P and tangent to the three spheres. Suppose plane Q intersects one of the spheres at a point A. Let D be the distance between A and the center of the equilateral triangle connecting the centers of the three spheres. Find $3D^2$.

29. Let ABC be an isosceles triangle with $AB = AC = 5$ and $BC = 6$. Let D be a point on the circumcircle of ABC such that AD is the diameter. If $BD = \frac{a}{b}$, where $\frac{a}{b}$ is in simplest form, find $a + b$.

30. How many permutations of $MATHTEAM$ contain the continuous string HAM but not $MEAT$?

31. Let p, q, r and s be the four roots of the quartic polynomial $x^4 - 67x^3 + 187x^2 + 108x + 2025$. Compute
$$\frac{2025}{pq} + \frac{2025}{pr} + \frac{2025}{ps} + \frac{2025}{qr} + \frac{2025}{qs} + \frac{2025}{rs}.$$

32. Suppose that c, h, j, and m are positive integers satisfying
$$j \times h \times m^2 + c = 100.$$
Determine the minimum value of
$$j + h + 2m + c.$$

33. In Dr. Abramson's logic class, everyone is either a truth teller or a liar. They each make the following statements:

    (a) Arnav: "If Daniel is a truth teller, Gabriele is a liar."

    (b) Bryan: "There are exactly 3 people lying."

    (c) Chris: "Arnav and I are the same kind of person."

    (d) Daniel: "Both Chris and Gabriele are truth tellers."

    (e) Fiona: "At least one of Bryan or Gabriele is a liar."

    (f) Gabriele: "Daniel is a liar."

    List the first letters of the names of the truth tellers in alphabetical order. For example, if everyone was telling the truth, the answer would be ABCDFG.

34. Define a sequence with $a_0 = 1$ and

$$a_n = a_{n-1} + 2a_{n-2} + \cdots + na_0.$$

    Given that all a_n are members of the Fibonacci sequence, find n such that $F_n = a_{2025}$. The Fibonacci sequence is defined as $F_0 = 1$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$.

35. How many ways are there to distribute 26 donuts among 4 children if no child gets more than 8 donuts?

36. I have an arithmetic sequence of integers $a_1, a_2, \ldots$ such that if I divide the sequence by 999, 1000, or 1001, every possible remainder comes up at least once. If the common difference of the terms is $d > 1$, find the minimum possible value for d.

37. In triangle ABC, $\overline{AB} = \overline{AC} = 10$, and $\overline{BC} = 12$. The angle bisector of $\angle ABC$ intersects the line through A parallel to $\overline{BC}$ at point D. Let point E be on $\overline{BC}$ such that $\angle ADB = \angle BDE$ and let point F be the intersection of $\overline{DE}$ and $\overline{AC}$. If $CF/AF = m/n$ in simplest form, find $m + n$.

38. BCA's lower cafeteria has 3 booths in which people can sit from $8:00$ AM to $4:00$ PM. At the beginning of each hour, each unoccupied booth has a $\frac{1}{2}$ chance of becoming occupied for the next two hours (all booths begin unoccupied at $8:00$ AM). The probability that Arnav finds an open booth at $10:30$ AM is $\frac{p}{q}$ where the fraction is in its simplest form. Find p.

39. Let $\triangle ABC$ be a triangle such that $AB = 13, BC = 14$, and $AC = 15$, and ω be the circumcircle of $\triangle ABC$. Let l be the line tangent to ω passing through C, and E be the intersection of l and the line perpendicular to l passing through A. If D is the foot of the altitude from A to $\overline{BC}$, $DE = \frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

40. Find the smallest value of $a + b + c + d$ given the following system of equations:

$$\begin{cases} a + b = 10, \\ ab + c + d = 33, \\ ad + bc = 44, \\ cd = 20. \end{cases}$$